Finding Limits section 2.2 Solutions

1.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	-0.1099	-0.111	-0.1111	?	-0.1111	-0.1112	-0.1124

$\lim_{x\to 2}\frac{x-2}{x^2-13x+22}\approx -01111$. The actual limit is -1/9

2.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	3.99334	3.99993	4.00000	4.00000	3.99993	3.99334

$\lim_{x\to 0}\frac{4\sin(x)}{x}\approx 4.00000$

3.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.3997	0.0400	0.0040	-0.0040	-0.0400	-0.3997

$\lim_{x\to 0}\frac{8\cos(x)-8}{x}\approx 0.00000$

4.

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$	0.2020	0.2002	0.2000	0.2000	0.1998	0.1980

$\lim_{x\to 5}\frac{\ln(x)-\ln 5}{x-5}\approx 0.2000$

5.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.1266	0.1252	0.1250	0.1250	0.1248	0.1235

$\lim_{x\to 1}\frac{x-4}{x^2+3x-28}\approx 0.1250$. The actual limit is 1/8

6.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	7.8333	8.9879	8.9999	8.9999	8.9879	7.8333

$\lim_{x\to 0}\frac{\sin(9x)}{x}\approx 8.9999$

7. $\lim_{x\to 1}\frac{|x-1|}{x-1}$ does not exist.

For values of x to the left of 2, $\lim_{x\to 1^-}\frac{|x-1|}{x-1}=-1$, whereas

for values of x to the right of 2, $\lim_{x\to 1^+}\frac{|x-1|}{x-1}=+1$

8. $\lim_{x\to 0} f(x)=\lim_{x\to 0}(x^2-2)=-2$

9. $\lim_{x\to 1}\sin(\pi x)=0$

10.

$\lim_{x\to -3}\frac{2}{x+3}$ does not exist because the function increases and decreases without bound as x approaches -3.

11. DNE

12. DNE

13.

(a) $f(1)$ exists. The black dot at (1, 2) indicates that $f(1) = 2$.

(b) $\lim_{x\to 1} f(x)$ does not exist. As x approaches 1 from the left, $f(x)$ approaches 5, whereas as x approaches 1 from the right, $f(x)$ approaches 1.

(c) $f(4)$ does not exist. The hollow circle at (4, 2) indicates that f is not defined at 4.

(d) $\lim_{x\to 4} f(x)$ exists. As x approaches 4, $f(x)$ approaches 2: $\lim_{x\to 4} f(x)=2$.

14.

(a) $f(2)$ does not exist. The vertical line indicates that f is not defined at −2.

(b) $\lim_{x \to -2} f(x) = 2$ does not exist. As x approaches −2, the values of $f(x)$ do not approach a specific number.

(c) $f(0)$ exists. The red dot at (0, 5) indicates that $f(0) = 5$.

(d) $\lim_{x \to 0} f(x)$ does not exist. As x approaches 0 from the left, $f(x)$ approaches 3.5, whereas as x approaches 0 from the right, $f(x)$ approaches 5.

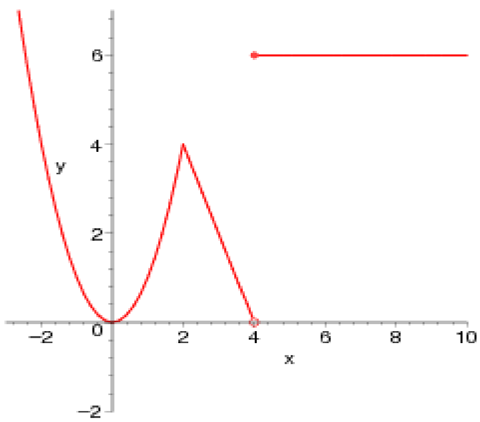
(e) $f(2)$ does not exist. The hollow circle at (2, 1/2) indicates that $f(2)$ is not defined.

(f) lim $x \to 2$ $f(x)$ exists. As x approaches 2, $f(x)$ approaches1/2 So $\lim_{x \to 2} f(x) = 1/2$

(g) $f(4)$ exists. The black dot at (4, 3) indicates that $f(4) = 3$.

(h) $\lim_{x \to 4} f(x)$ does not exist. As x approaches 4, the values of $f(x)$ do not approach a specific number.

15.

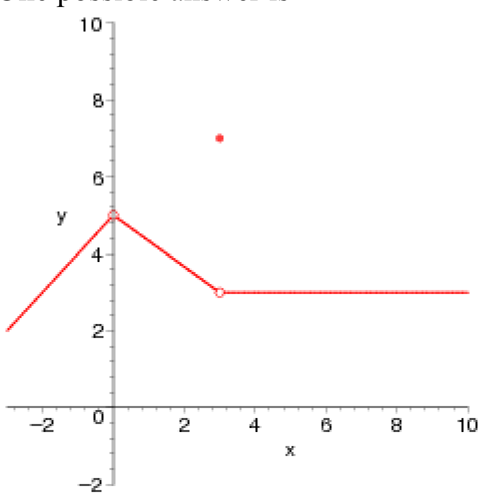


a-

(ii) is the correct choice: $\lim_{x \to c} f(x)$ exists for all values of $c \neq 4$.

16.

One possible answer is



Finding limits section 2.2

1. Consider the following limit. $\lim_{x \to 2} \frac{x-2}{x^2 - 13x + 22}$

a-Complete the table. (Round your answers to four decimal places.)

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

b-Use the result to estimate the limit.

2. Consider the following limit. $\lim_{x \to 0} \frac{4\sin(x)}{x}$

a-Complete the table. (Round your answers to five decimal places.)

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

b-Use the result to estimate the limit.

3.Complete the table and use the result to estimate the limit

$$\lim_{x \to 0} \frac{8\cos(x) - 8}{x}$$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

4. Complete the table and use the result to estimate the limit. $\lim_{x \to 5} \frac{\ln(x) - \ln 5}{x-5}$

x	4.9	4.99	4.999	5	5.001	5.01	5.1
$f(x)$				?			

5. Consider the following. $\lim_{x \to 1} \frac{x-4}{x^2 + 3x - 28}$

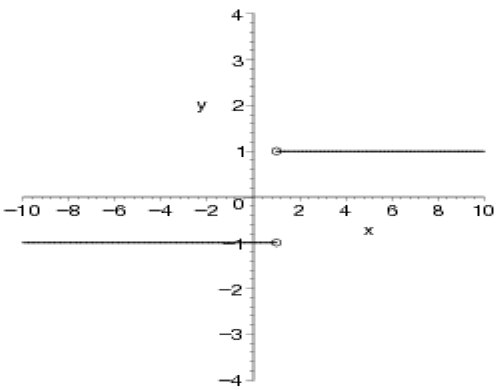
Complete the table and use the result to estimate the limit

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$				?			

6. Complete the table and use the result to estimate the limit given $\lim_{x\to 0}\frac{\sin(9x)}{x}$

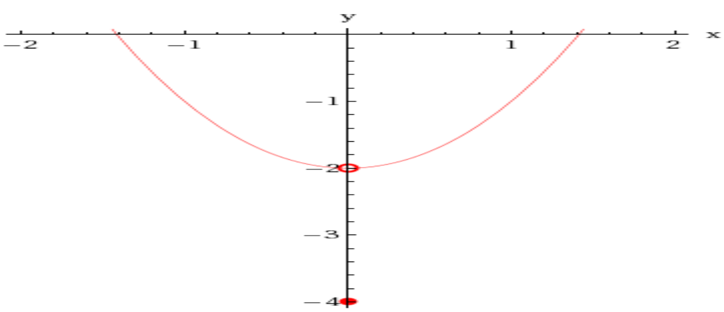
x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

7. Consider the following. $\lim_{x\to 1}\frac{|x-1|}{x-1}$ Use the graph to find the limit (if it exists). (If an answer does not exist, write DNE.)



8. Consider the following.

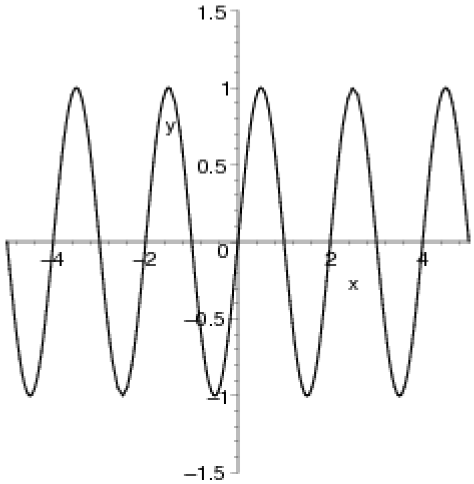
$$f(x)=\begin{cases} x^2-2 & if\ x\neq 0 \\ -4 & if\quad x=0 \end{cases}$$



Use the graph to find the limit below (if it exists). (If an answer does not exist, write DNE.)

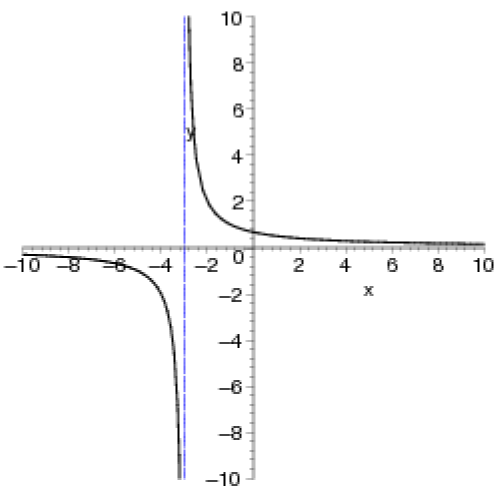
9. Consider the following.

$\lim_{x\to 1}\sin(\pi x)$



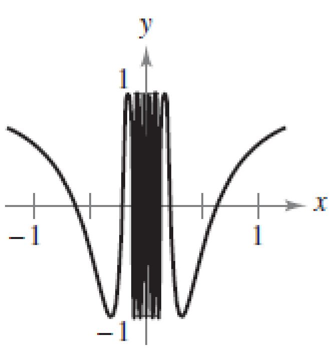
Use the graph to find the limit (if it exists). (If an answer does not exist, write DNE.)

10. Consider the following. $\lim_{x \to -3} \frac{2}{x+3}$

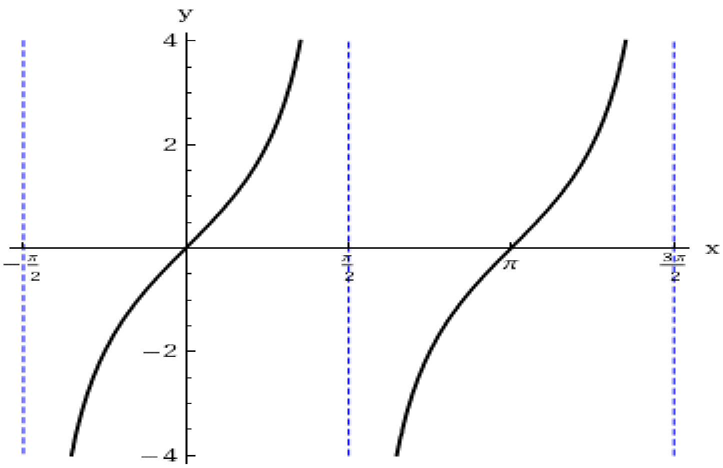


Use the graph to find the limit (if it exists). (If an answer does not exist, write DNE.)

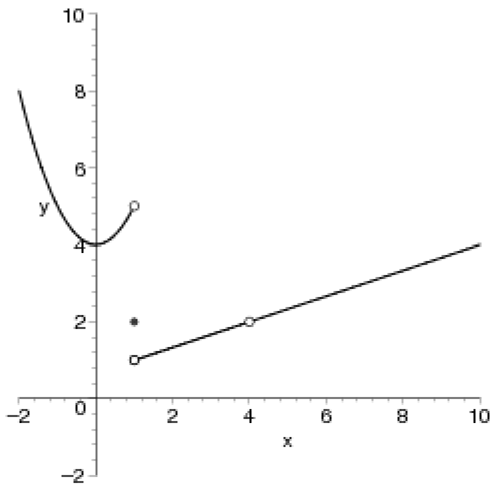
11. Use the graph to find the limit (if it exists). (If an answer does not exist, write DNE.) $\lim_{x \to 0} \cos \frac{1}{x}$



12. Use the graph to find the limit (if it exists). (If an answer does not exist, write DNE.) $\lim_{x \to \pi/2} 2 \tan x$



13. Use the graph of the function *f* to decide whether the value of the given quantity exists. If it does, find it. If it does not, write DNE.



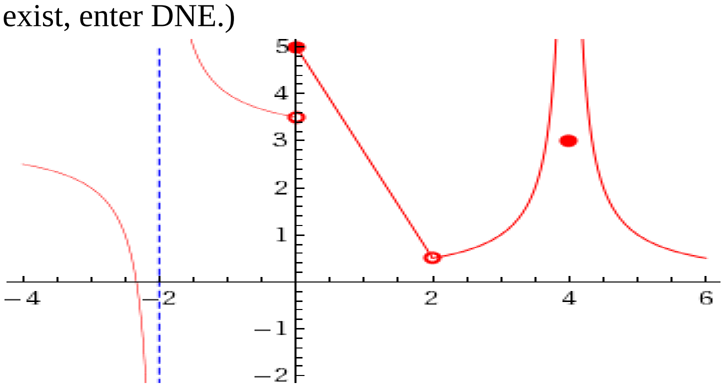
(a) $f(1)$

(b) $\lim_{x \to 1} f(x)$

(c) $f(4)$

(d) $\lim_{x \to 4} f(x)$

14.
Use the graph of the function *f* to decide whether the value of the given quantity exists. (If an answer does not exist, enter DNE.)



(a) $f(2)$

(b) $\lim_{x \to -2} f(x)$

(c) $f(0)$

(d) $\lim_{x \to 0} f(x)$

(e) $f(2)$

(f) $\lim_{x \to 2} f(x)$

(g) $f(4)$

(h) $\lim_{x \to 4} f(x)$

15. Consider the following.

$$f(x) = \begin{cases} x^2, & x \le 2 \\ 8 - 2x, & 2 < x < 4 \\ 6, & x \ge 4 \end{cases}$$

a-Sketch the graph of *f*.
b-Identify the values of *c* for which the following limit exists. $\lim_{x \to c} f(x)$

(i) The limit exists at all points on the graph except where $c = 2$ and $c = 4$.
(ii) The limit exists at all points on the graph except where $c = 4$
(iii) The limit exists at all points on the graph except where $c = 2$.
(iv) The limit exists at all points on the graph.

16. Sketch a graph of a function *f* that satisfies the given values.
$f(0)$ is undefined
$\lim_{x \to 0} f(x) = 5$; $\lim_{x \to 3} f(x) = 3$; $f(3)=7$

2.3 Evaluating Limits Analytically and L'hopital rule

1-Find the limit.
$$\lim_{x \to -4} x^2$$

2-Find the limit.
$$\lim_{x \to 3} x^3$$

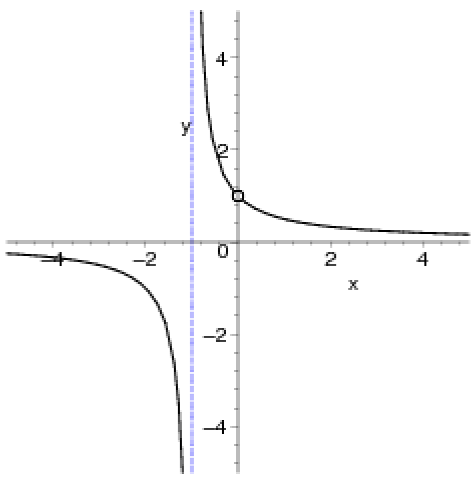
3-Find the limit. $\lim_{x \to 2} \sqrt{x+7}$

4-Find the limit.
$$\lim_{x \to 3} \frac{\sqrt{x}+1}{x-2}$$

5-Find the limit of the trigonometric function. $\lim_{x \to \pi} 5 \sin x$

6-Find the limit of the trigonometric function. $\lim_{x \to 0} 5 \tan x$

7-
Consider the following function and its graph. $f(x) = \dfrac{x}{x^2 + x}$



Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim_{x \to -1} f(x)$

(b) $\lim_{x \to 0} f(x)$

(c) Write a simpler function that agrees with the given function at all but one point.

8-

(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x\to -7} \frac{x^2-49}{x+7}$$

(b)Write a simpler function that agrees with the given function at all but one point.

9-
(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x\to -3} \frac{2x^2-2x-24}{x+3}$$

(b)Write a simpler function that agrees with the given function at all but one point.
10-
(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x\to 3} \frac{x^3-27}{x-3}$$

(b)Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

11-
(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x\to \ln 2} \frac{e^{2x}-4}{e^x-2}$$

(b)Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

12-
(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x\to 3} \frac{x^3-27}{x-3}$$

(b)Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.

13-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$$

14-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to \pi/2} \frac{8 \cos x}{\cot x}$$

15-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{t \to 0} \frac{\sin 4t}{3t}$$

16-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{t \to 0} \frac{\sin t}{7t}$$

17-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{t \to 0} \frac{\sin^8 t}{t}$$

18-Evaluate each limit using l'Hopital rule. Note that if $\lim_{x \to \pm\infty} f(x) = c$ then y = c is called a horizontal asymptote.

a-. $\lim_{x \to 0} \frac{\sin 5x}{x^2 + 6x}$

b- $\lim_{x \to 0} \frac{6x^3}{\sin x - x}$

c- $\lim_{x \to 0} \frac{\cos 5x - 1}{\sin 6x}$

d- $\lim_{x \to \infty} \frac{7x + 4}{3 - 4x}$

For instance, here, y=-7/4 is a horizontal asymptote

e- $\lim_{x \to -\infty} 7x \sin\left(\frac{1}{x}\right)$

f- $\lim_{x\to\infty}\left(\frac{\ln 7x}{12x^{1/2}}\right)$

g- $\lim_{x\to\infty}\left(\frac{7x}{e^x}\right)$

h- $\lim_{x\to\infty}\left(\frac{x^2}{e^x}\right)$

i- $\lim_{x\to 1}\left(\frac{\sqrt{24+x}-5x^{1/4}}{x^2-7x+6}\right)$

j- $\lim_{x\to 16}\left(\frac{1}{\sqrt{x}-4}-\frac{8}{x-16}\right)$

k- $\lim_{x\to 0^+}\left(8x^{\sin 6x}\right)$

l- $\lim_{x\to 0}\left(\frac{\sin 16x}{\sin 26x}\right)$

m- $\lim_{x\to 0}\left(\frac{\sin(16x)-8x\cos(8x)}{8x-\sin(8x)}\right)$

n- $\lim_{x\to\pi/2}\frac{\cos x}{\sin(x+\frac{\pi}{2})}$

o- $\lim_{x\to 0}\frac{6x^2}{\cos x-1}$

p- $\lim_{x\to\pi/2}(\sec x-\tan x)$

q- $\lim_{x\to 11}\left(\tan(\frac{\pi}{2}x)\ln(\frac{x}{11})\right)$

r- $\lim_{x\to 0}\frac{6^x-7^x}{x}$

s- $\lim_{x\to\infty}x^{6/x}$

t- $\lim_{x\to 1}\left(1 + 6\ln x\right)^{8/(x-1)}$

u- $\lim_{x\to\infty}\left(\mathrm{e}^{-x}(x^3 - x^2 + 6)\right)$

v- $\lim_{x\to\pi/2}\left(\frac{e^{3x} + 3x - 1}{2\ln(x+1)}\right)$

w- $\lim_{x\to 0}\frac{5\cos x - 1}{x}$

2.3 Evaluating Limits Analytically and L'hopital rule Solutions

1-Find the limit.
$\lim_{x \to -4} x^2$

Ans: 16

2-Find the limit.
$\lim_{x \to 8} x^3$

Ans: 512

3-Find the limit. $\lim_{x \to 2} \sqrt{x+7}$

Ans: 3

4-Find the limit.
$\lim_{x \to 8} \frac{\sqrt[3]{x}+1}{x-2}$
Ans: 1/2

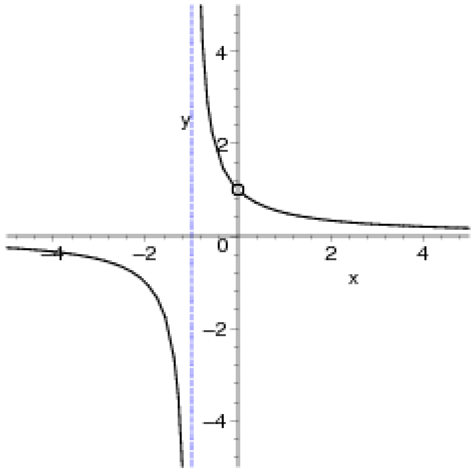
5-Find the limit of the trigonometric function. $\lim_{x \to \pi} 5 \sin x$

Ans: 0

6-Find the limit of the trigonometric function. $\lim_{x \to 0} 5 \tan x$

Ans: 0

7-
Consider the following function and its graph. $f(x) = \frac{x}{x^2 + x}$



Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim_{x \to -1} f(x)$

Ans: DNE

(b) $\lim_{x \to 0} f(x)$

Ans:1

(c) Write a simpler function that agrees with the given function at all but one point.

Ans: $g(x) = \dfrac{1}{x+1}$

8-

(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to -7} \frac{x^2 - 49}{x+7}$$

Ans: -14

(b)Write a simpler function that agrees with the given function at all but one point.

Ans: $g(x) = x - 7$

9-

(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to -3} \frac{2x^2 - 2x - 24}{x+3}$$

Ans: -14

(b)Write a simpler function that agrees with the given function at all but one point.

Ans: $g(x) = 2x - 8$

10-

(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$
Ans: 27

(b)Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.
Ans: $g(x) = x^3 + 3x + 9$

11-
(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to \ln 2} \frac{e^{2x} - 4}{e^x - 2}$$
Ans: 4

(b)Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.
Ans: $g(x) = e^x + 2$

12-
(a)Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$
Ans: 27

(b)Write a simpler function that agrees with the given function at all but one point. Use a graphing utility to confirm your result.
Ans: $g(x) = x^3 + 3x + 9$

13-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 0} \frac{\sqrt{x+6} - \sqrt{6}}{x}$$
Ans: $\dfrac{\sqrt{6}}{12}$

14-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to \pi/2} \frac{8\cos x}{\cot x}$$
Ans: 8

15-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{t\to 0}\frac{\sin 4t}{3t}$$

Ans: 4/3

16-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{t\to 0}\frac{\sin t}{7t}$$

Ans: 1/7

17-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{t\to 0}\frac{\sin^8 t}{t}$$

Ans: 0

18-Evaluate each limit using l'Hopital rule. Note that if $\lim\limits_{x\to\pm\infty} f(x)=c$ then *y* = c is called a horizontal asymptote.

a-. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x^2+6x}$

Ans: $\frac{5}{6}$

b- $\lim\limits_{x\to 0}\dfrac{6x^3}{\sin x - x}$

Ans: -36

c- $\lim\limits_{x\to 0}\dfrac{\cos 5x - 1}{\sin 6x}$

Ans: 0

d- $\lim\limits_{x\to\infty}\dfrac{7x+4}{3-4x}$

Ans: -7/4

For instance, here, y=-7/4 is a horizontal asymptote

e- $\lim\limits_{x\to-\infty} 7x\sin\left(\dfrac{1}{x}\right)$

Ans: 7

f- $\lim_{x\to\infty}\left(\frac{\ln 7x}{12x^{1/2}}\right)$
Ans: 0

g- $\lim_{x\to\infty}\left(\frac{7x}{e^x}\right)$
Ans: 0

h- $\lim_{x\to\infty}\left(\frac{x^2}{e^x}\right)$
Ans: 0

i- $\lim_{x\to 1}\left(\frac{\sqrt{24+x}-5x^{1/4}}{x^2-7x+6}\right)$

Ans: $\frac{23}{100}$

j- $\lim_{x\to 16}\left(\frac{1}{\sqrt{x}-4}-\frac{8}{x-16}\right)$

Ans: $\frac{1}{8}$

k- $\lim_{x\to 0^+}\left(8x^{\sin 6x}\right)$

Ans: 1

l- $\lim_{x\to 0}\left(\frac{\sin 16x}{\sin 26x}\right)$

Ans: $\frac{8}{13}$

m- $\lim_{x\to 0}\left(\frac{\sin(16x)-8x\cos(8x)}{8x-\sin(8x)}\right)$
Ans: 2

n- $\lim_{x\to \pi/2}\frac{\cos x}{\sin(x+\frac{\pi}{2})}$

Ans: 1

o- $\lim_{x\to 0}\dfrac{6x^2}{\cos x - 1}$

Ans: -12

p- $\lim_{x\to \pi/2}(\sec x - \tan x)$

Ans: 0

q- $\lim_{x\to 11}\left(\tan(\dfrac{\pi}{2}x)\ln(\dfrac{x}{11})\right)$

Ans: $-\dfrac{2}{11\pi}$

r- $\lim_{x\to 0}\dfrac{6^x - 7^x}{x}$

Ans: $\ln\left(\dfrac{6}{7}\right)$

s- $\lim_{x\to \infty} x^{6/x}$

Ans: 1

t- $\lim_{x\to 1}\left(1 + 6\ln x\right)^{8/(x-1)}$

Ans: e^{48}

u- $\lim_{x\to \infty}\left(\mathrm{e}^{-x}(x^3 - x^2 + 6)\right)$

Ans: 0

v- $\lim_{x\to \pi/2}(\dfrac{e^{3x} + 3x - 1}{2\ln(x+1)})$

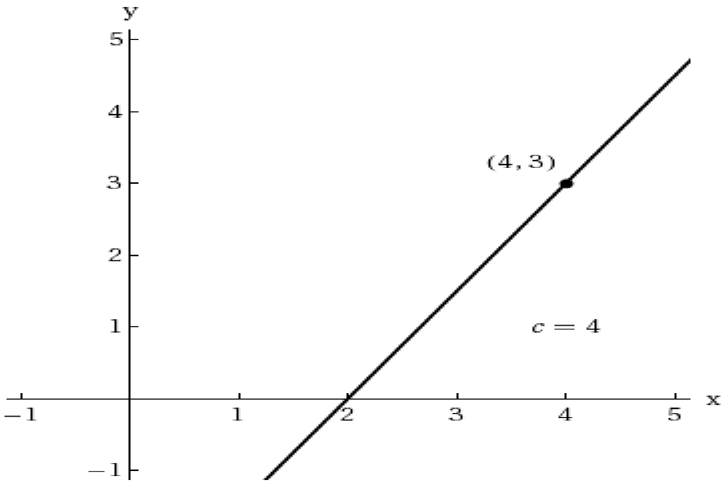
Ans: 3

w- $\lim_{x\to 0}\dfrac{5\cos x - 1}{x}$

Ans: 0

2.4 Continuity and One-Sided limits

1-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

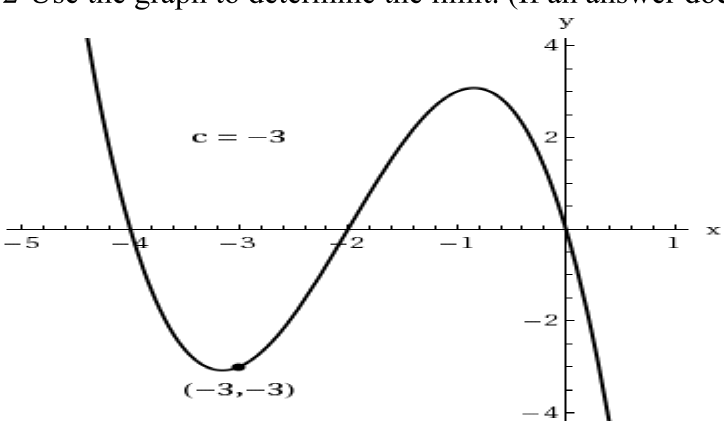
(a) $\lim_{x \to c^+} f(x)$

(b) $\lim_{x \to c^-} f(x)$

(c) $\lim_{x \to c} f(x)$

(ii) Is the function continuous at $x = 4$?

2-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

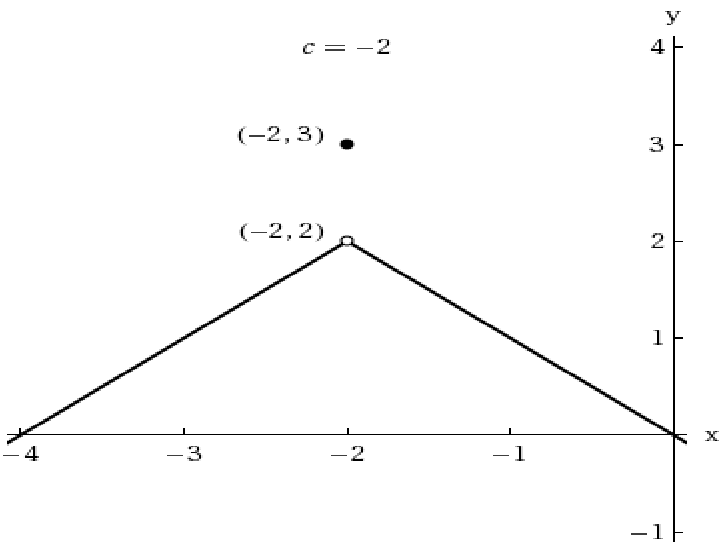
(a) $\lim_{x \to c^+} f(x)$

(b) $\lim_{x \to c^-} f(x)$

(c) $\lim_{x \to c} f(x)$

(ii) Is the function continuous at $x = -3$?

3-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

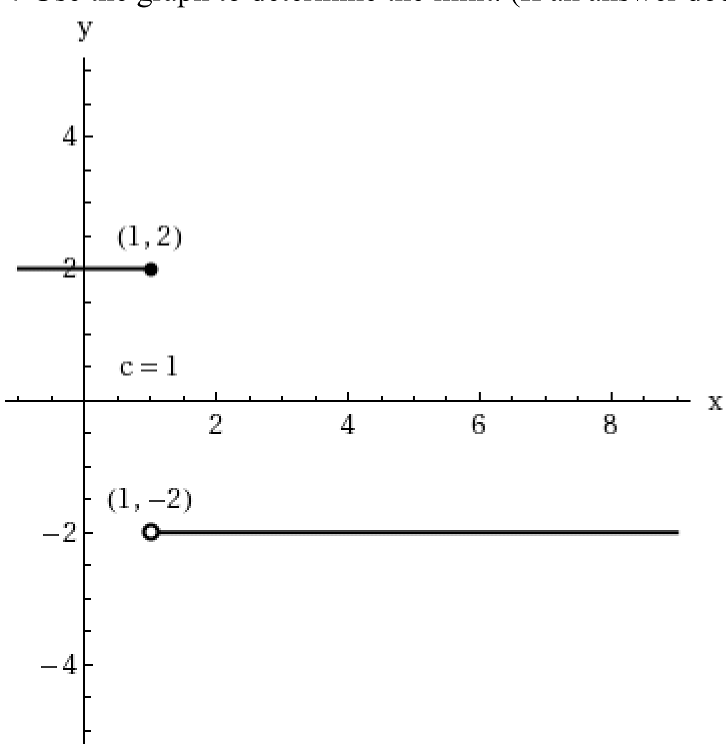
(a) $\lim_{x \to c^+} f(x)$

(b) $\lim_{x \to c^-} f(x)$

(c) $\lim_{x \to c} f(x)$

(ii) Is the function continuous at x = -2?

4-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim_{x \to c^+} f(x)$

(b) $\lim_{x \to c^-} f(x)$

(c) $\lim_{x \to c} f(x)$

(ii) Is the function continuous at $x = 1$?

Use some approximations to determine the limits in each of the following problems: 5-8

5-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x \to -2^-} \frac{x}{\sqrt{x^2 - 4}}$

6-Find the limit (if it exists). (If an answer does not exist, enter DNE.)

$\lim_{x \to 2^+} \frac{|x-2|}{x-2}$

7-Find the limit (if it exists). (If an answer does not exist, enter DNE.)

$\lim_{x \to \pi} 2 \cot x$

8-Find the limit (if it exists).

$\lim_{x \to 2^+} \ln(x-2)$

9-Find the constant a such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} 4x^2, & x \ge 1 \\ ax - 8, & x < 1 \end{cases}$$

10-Find the constants a and b such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} 5 & x \le -2 \\ ax + b & -2 < x < 3 \\ -5 & x \ge 3 \end{cases}$$

11-Consider the following function $g(x) = \begin{cases} \frac{x^2 - a^2}{x - a}, & x \ne a \\ 6, & x = a \end{cases}$

Find the constant a such that the function is continuous on the entire real line.

12-Find the constant a such that the function is continuous on the entire real line. $f(x) = \begin{cases} \frac{4 \sin x}{x}, & x < 0 \\ a - 9x, & x \ge 0 \end{cases}$

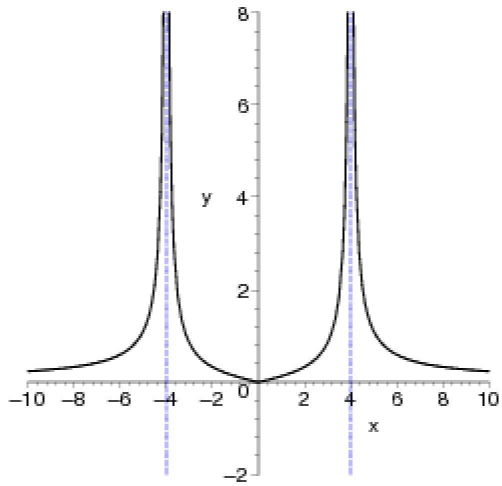
13-Find the constant a such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} 2e^{ax} - 2, & x \le 4 \\ \ln(x-3) + x^2, & x > 4 \end{cases}$$

14-Find all values of c such that f is continuous on $(-\infty, \infty)$.

$$f(x)=\begin{cases}7-x^2, & x\le c\\ x, & x>c\end{cases}$$

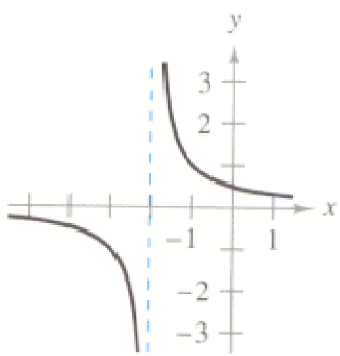
15-Use the graph to determine the limit. (If an answer does not exist, enter DNE.) $f(x)=2\left|\dfrac{x}{x^2-16}\right|$



(a) $\lim\limits_{x\to 4^+} f(x)$

(b) $\lim\limits_{x\to 4^-} f(x)$

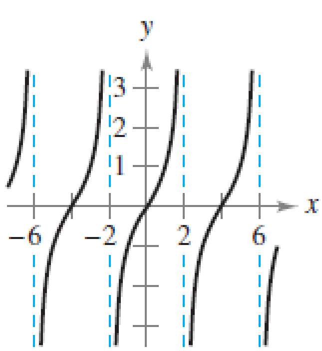
16-Consider the following function and graph. $f(x)=\dfrac{1}{x+2}$



(a) $\lim\limits_{x\to -2^+} f(x)$

(b) $\lim\limits_{x\to -2^-} f(x)$

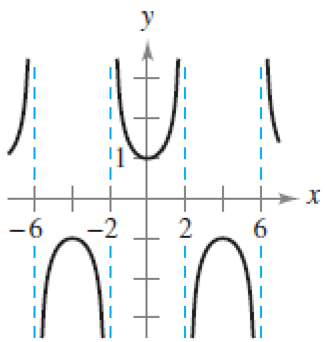
17-Consider the following function and graph. $f(x)=\tan\dfrac{\pi x}{4}$



(a) $\lim_{x\to -2^+} f(x)$

(b) $\lim_{x\to -2^-} f(x)$

18-Consider the following function and graph. $f(x) = \sec\dfrac{\pi x}{4}$



(a) $\lim_{x\to -2^+} f(x)$

(b) $\lim_{x\to -2^-} f(x)$

19-Consider the following limit. $f(x) = \dfrac{1}{x^2 - 16}$

a-Complete the table. (Round your answers to two decimal places.)

x	-4.5	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9	-3.5
$f(x)$					?				

b-Use the table to determine

(i) $\lim_{x\to -4^-} f(x)$

(ii) $\lim_{x\to -4^+} f(x)$

(iii) $\lim_{x\to -4} f(x)$

20-Consider the following limit. $f(x) = \cot\dfrac{\pi x}{4}$

a-Complete the table. (Round your answers to two decimal places.)

x	-4.5	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9	-3.5
$f(x)$					?				

b-Use the table to determine

(i) $\lim_{x\to -4^-} f(x)$

(ii) $\lim_{x\to -4^+} f(x)$

(iii) $\lim_{x\to -4} f(x)$

- **Note that if** $f(\mathrm{c}) = \mathit{undefined}$ **and** $\lim_{x\to c^{\pm}} f(x) = \pm\infty$ **then x = c is called a vertical asymptote.**

21-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $f(x) = \dfrac{5}{x^2}$

22-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $g(x) = \dfrac{t-5}{t^2+25}$

23-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $f(x) = \dfrac{e^{-2x}}{x-5}$

24-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $g(x) = xe^{-5x}$

25-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $f(z) = \ln(z^2 - 25)$

26-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $f(x) = \ln(x + 5)$

27-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $f(x) = 5\tan(\pi x)$

28-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.) $s(t) = \dfrac{5t}{\sin(t)}$

29-Find the one-sided limit (if it exists). (If the limit does not exist, enter DNE.) $\lim_{x\to 5^-} \dfrac{1}{(x-5)^2}$

30-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x\to -\left(\frac{1}{2}\right)^+} \dfrac{6x^2+x-1}{4x^2-4x-3}$

31-Find the one-sided limit (if it exists). (If the limit does not exist, enter DNE.) $\lim_{x\to 0^-} \left(7+\dfrac{8}{x}\right)$

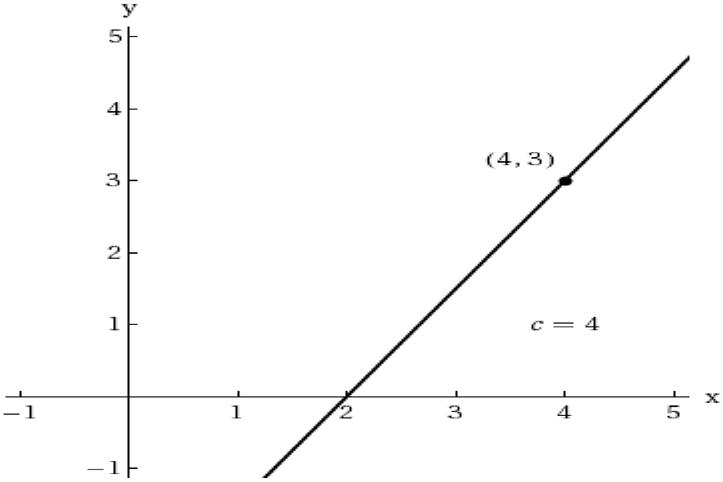
32-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x\to 0^+} \dfrac{5}{\sin x}$

33-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x\to (\pi/2)^+} \dfrac{-5}{\cos x}$

34-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x \to 8^+} \ln(x^2 - 64)$

2.4 Continuity and One-Sided limits solutions

1-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim\limits_{x \to c^+} f(x)$

Ans: 3

(b) $\lim\limits_{x \to c^-} f(x)$

Ans:3

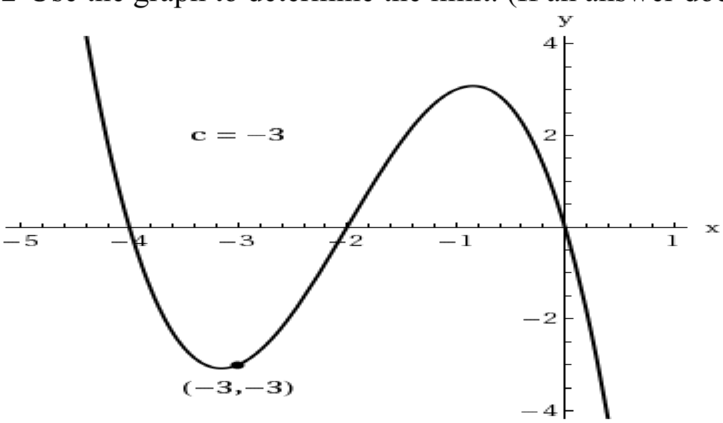
(c) $\lim\limits_{x \to c} f(x)$

Ans: 3

(ii) Is the function continuous at $x = 4$?

Ans: Yes

2-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim\limits_{x \to c^+} f(x)$

Ans: -3

(b) $\lim\limits_{x \to c^-} f(x)$

Ans:-3

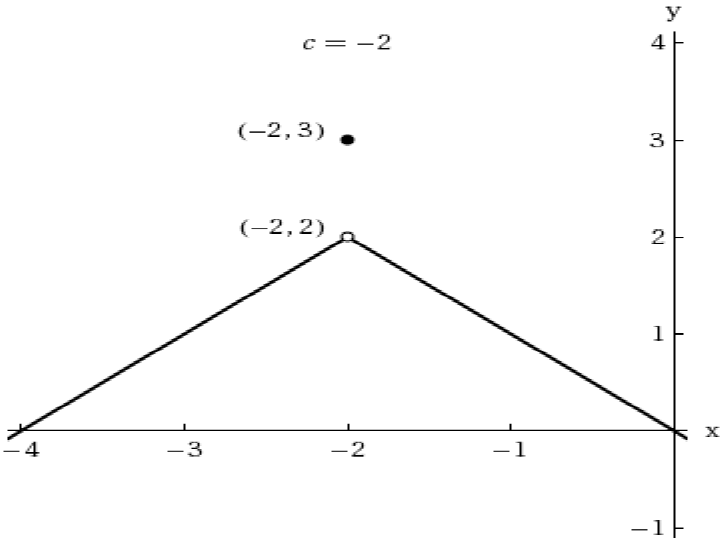
(c) $\lim_{x \to c} f(x)$

Ans: -3

(ii) Is the function continuous at $x = -3$?

Ans: Yes

3-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim_{x \to c^+} f(x)$

Ans: 2

(b) $\lim_{x \to c^-} f(x)$

Ans: 2

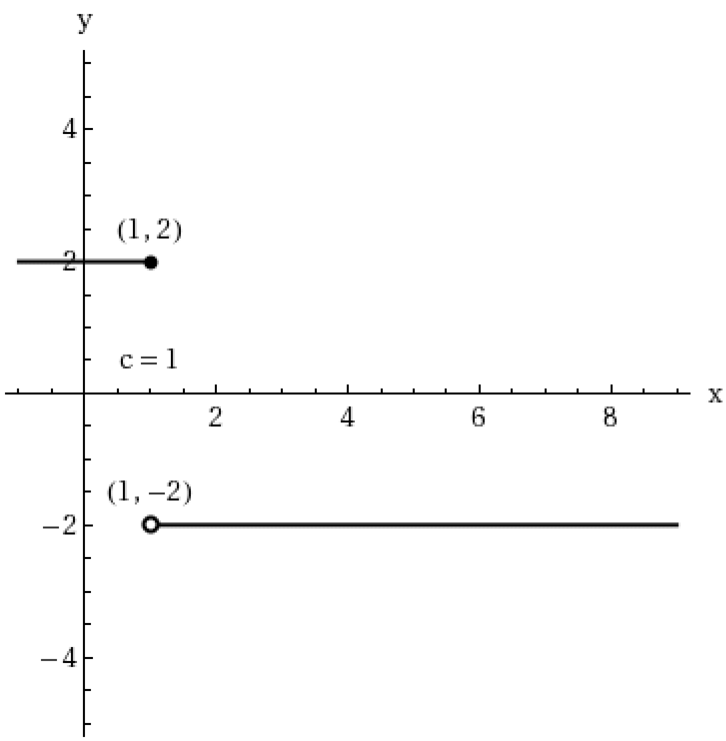
(c) $\lim_{x \to c} f(x)$

Ans: 3

(ii) Is the function continuous at $x = -2$?

Ans: No

4-Use the graph to determine the limit. (If an answer does not exist, enter DNE.)



(i)Use the graph to determine the limit visually (if it exists). (If an answer does not exist, enter DNE.)

(a) $\lim\limits_{x \to c^+} f(x)$

Ans: -2

(b) $\lim\limits_{x \to c^-} f(x)$

Ans: 2

(c) $\lim\limits_{x \to c} f(x)$

Ans: 2

(ii) Is the function continuous at $x = 1$?

Ans: No

Use some approximations to determine the limits in each of the following problems: 5-8

5-Find the limit of the function (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to -2^-} \frac{x}{\sqrt{x^2 - 4}}$$

Ans: $-\infty$

6-Find the limit (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 2^+} \frac{|x - 2|}{x - 2}$$

Ans: 1

7-Find the limit (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to \pi} 2 \cot x$$

Ans: DNE (b/c left-sided limit is $-\infty$ while the right-sided limit is $+\infty$)

8-Find the limit (if it exists).

$$\lim_{x \to 2^+} \ln(x-2)$$

Ans: $-\infty$

9-Find the constant *a* such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} 4x^2, & x \geq 1 \\ ax - 8, & x < 1 \end{cases}$$

Ans: a=12

10-Find the constants *a* and *b* such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} 5, & x \leq -2 \\ ax + b, & -2 < x < 3 \\ -5, & x \geq 3 \end{cases}$$

Ans: a=-2, b=1

11-Consider the following.

$$g(x) = \begin{cases} \frac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 6 & \text{if } x = a \end{cases}$$

Find the constant *a* such that the function is continuous on the entire real line.

Ans: a=3

12-Find the constant *a* such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} \dfrac{4\sin x}{x}, & x < 0 \\ a - 9x, & x \geq 0 \end{cases}$$

Ans: a=4

13-

Find the constant *a* such that the function is continuous on the entire real number line.

$$f(x) = \begin{cases} 2e^{ax} - 2, & x \leq 4 \\ \ln(x-3) + x^2, & x > 4 \end{cases}$$

Ans $a = \dfrac{\ln 3}{2}$

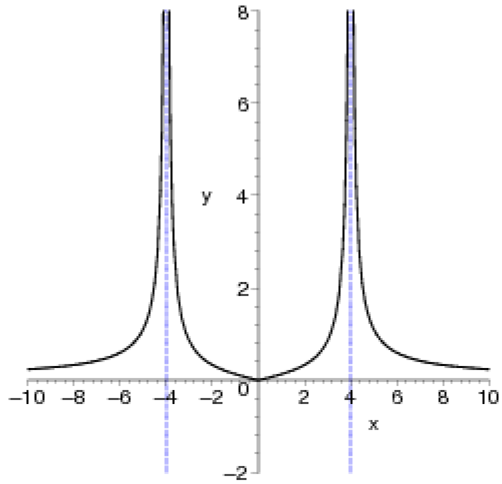
14-Find all values of *c* such that *f* is continuous on (-∞, ∞).

$$f(x) = \begin{cases} 7 - x^2, & x \leq c \\ x, & x > c \end{cases}$$

$c =$

$$-\frac{1}{2} - \frac{\sqrt{29}}{2}; \quad -\frac{1}{2} + \frac{\sqrt{29}}{2}$$

15-Use the graph to determine the limit. (If an answer does not exist, enter DNE.) $f(x) = 2\left|\frac{x}{x^2 - 16}\right|$



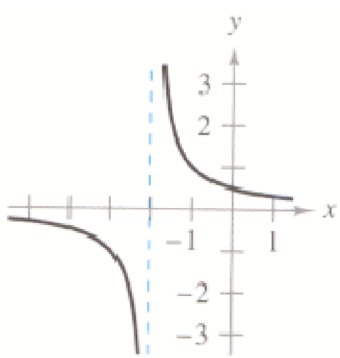
(a) $\lim_{x \to 4^+} f(x)$

Ans: ∞

(b) $\lim_{x \to 4^-} f(x)$

Ans:∞

16-Consider the following function and graph. $f(x) = \frac{1}{x+2}$



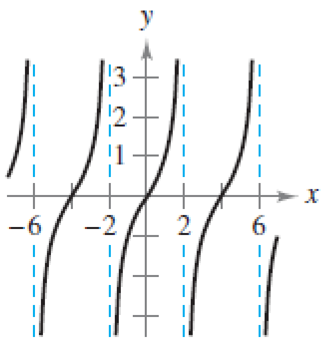
(a) $\lim_{x \to -2^+} f(x)$

Ans: ∞

(b) $\lim_{x \to -2^-} f(x)$

Ans:-∞

17-Consider the following function and graph. $f(x) = \tan \frac{\pi x}{4}$



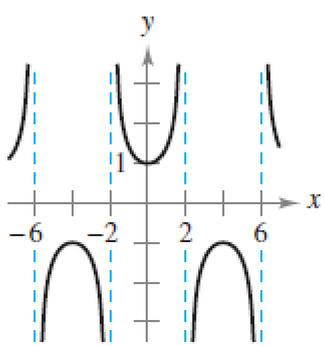
(a) $\lim_{x \to -2^+} f(x)$

Ans: -∞

(b) $\lim_{x \to -2^-} f(x)$

Ans:+∞

18-Consider the following function and graph. $f(x) = \sec\frac{\pi x}{4}$



(a) $\lim_{x \to -2^+} f(x)$

Ans: ∞

(b) $\lim_{x \to -2^-} f(x)$

Ans:-∞

19-Consider the following limit. $f(x) = \frac{1}{x^2 - 16}$

a-Complete the table. (Round your answers to two decimal places.)

x	-4.5	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9	-3.5
$f(x)$	0.24	1.23	12.48	124.98	?	-125.02	-12.52	-1.27	-0.27

b-Use the table to determine

(i) $\lim_{x \to -4^-} f(x)$

Ans: ∞

(ii) $\lim_{x \to -4^+} f(x)$

Ans:-∞

(iii) $\lim_{x \to -4} f(x)$

Ans: DNE

20-Consider the following limit. $f(x) = \cot \frac{\pi x}{4}$

a-Complete the table. (Round your answers to two decimal places.)

x	-4.5	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9	-3.5
$f(x)$	-2.41	-12.71	-127.32	-1273.24	?	1273.24	127.32	12.71	2.41

b-Use the table to determine

(i) $\lim_{x \to -4^-} f(x)$

Ans: -∞

(ii) $\lim_{x \to -4^+} f(x)$

Ans:+∞

(iii) $\lim_{x \to -4} f(x)$

Ans: DNE

- **Note that if $f(c) = undefined$ and $\lim_{x \to c} f(x) = \pm\infty$ then $x = c$ is called a vertical asymptote.**

21-Find the vertical asymptotes (if any) of the graph of the function. (Use *n* as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)

$f(x) = \frac{5}{x^2}$

Ans: $x = 0$

22-Find the vertical asymptotes (if any) of the graph of the function. (Use *n* as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)

$g(x) = \frac{t-5}{t^2+25}$

Ans : DNE

23-Find the vertical asymptotes (if any) of the graph of the function. (Use *n* as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)

$f(x) = \frac{e^{-2x}}{x-5}$

Ans : $x = 5$

24-Find the vertical asymptotes (if any) of the graph of the function. (Use *n* as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)

$g(x) = xe^{-5x}$

Ans: DNE

25-Find the vertical asymptotes (if any) of the graph of the function. (Use *n* as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)

$f(z) = \ln(z^2 - 25)$

Ans: z=-5, 5

26-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)
$f(x) = \ln(x + 5)$
Ans: $x = -5$

27-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)
$f(x) = 5\tan(\pi x)$
Ans: $x = \frac{1}{2} + n$

28-Find the vertical asymptotes (if any) of the graph of the function. (Use n as an arbitrary integer if necessary. If an answer does not exist, enter DNE.)

$$s(t) = \frac{5t}{\sin(t)}$$

$t = n \cdot \pi$ where $n \neq 0$

29-Find the one-sided limit (if it exists). (If the limit does not exist, enter DNE.)

$$\lim_{x \to 5^-} \frac{1}{(x-5)^2}$$

Ans: -∞

30-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.)

$$\lim_{x \to -\left(\frac{1}{2}\right)^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3}$$

Ans: 5/8

31-Find the one-sided limit (if it exists). (If the limit does not exist, enter DNE.)

$$\lim_{x \to 0^-} \left(7 + \frac{8}{x}\right)$$

Ans: $-\infty$

32-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x \to 0^+} \frac{5}{\sin x}$

Ans: ∞

33-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x \to (\pi/2)^+} \frac{-5}{\cos x}$

Ans: ∞

34-Find the one-sided limit (if it exists). (If an answer does not exist, enter DNE.) $\lim_{x \to 8^+} \ln(x^2 - 64)$

Ans: -∞

3.2 Derivative: Power Rule

Apply the Power Rule to compute the derivative.

1- $\dfrac{d}{dx}x^{13/3}$

2- $\dfrac{d}{dt}t^{\sqrt{3}}$

Apply the Power Rule to compute the derivative.

3- $\dfrac{d}{dx}\left(6x^4\right)\Big|_{x=-2}$

4- $\dfrac{d}{dt}\left(t^{-5}\right)\Big|_{t=4}$

5- $\dfrac{d}{dt}\left(t^{-4/5}\right)\Big|_{t=1}$

Find an equation of the tangent line to the graph $f(x)$ at $x = a$.

6- $f(x) = 2x^4$, $a = 2$

7- $f(x) = 4x - 32\sqrt{x}$, $a = 4$

8- $f(x) = 5\sqrt[3]{x}$, $a = 27$

Calculate the derivative of each given function, ie find y'

9- $y=9x^3 - 3x^2 + 2$

10- $y=4x^3 + 4x^2 + 2x$

11- $y= 2x^{5/3} - 8x^{-2} - 12$

12- $y= 7y^4 + 4y^{4/5}$

13- $y=3e^4 + 2x^7$

14- $y=6e^x + 4x^3$

3.2 Derivative: Power Rule solutions

Apply the Power Rule to compute the derivative.

1- $\frac{d}{dx} x^{13/3}$

Ans: $\frac{13}{3} x^{\frac{10}{3}}$

2- $\frac{d}{dt} t^{\sqrt{3}}$

Ans: $\sqrt{3}\, t^{\sqrt{3}-1}$

Apply the Power Rule to compute the derivative.

3- $\frac{d}{dx}\left(6x^4\right)\Big|_{x=-2}$

Ans: derivative is $24x^3$ at x=-2 gives -192

4- $\frac{d}{dt}\left(t^{-5}\right)\Big|_{t=4}$

Ans: derivative is $-5t^{-6}$ at t=-2, gives -5/4096

5- $\frac{d}{dt}\left(t^{-4/5}\right)\Big|_{t=1}$

Ans: -4/5

Find an equation of the tangent line to the graph $f(x)$ at $x = a$.

6- $f(x) = 2x^4$, $a = 2$

Ans: Let $f(x) = 2x^4$. Then, by the Power Rule, $f'(x) = 8x^3$. The equation of the tangent line to the graph of $f(x)$ at $x = 2$ is $y = f'(2)(x - 2) + f(2) = 64(x - 2) + 32 = 64x - 96$

7- $f(x) = 4x - 32\sqrt{x}$, $a = 4$
Ans: Let $f(x) = 4x - 32x^{1/2}$.Then $f'(x) = 4 - 16x^{-1/2}$.In particular, $f'(4) = -4$.The tangent line at $x = 4$ is $y = f'(4)(x - 4) + f(4) = -4(x - 4) + (-48) = -4x - 32$

8- $f(x) = 5\sqrt[3]{x}$, $a = 27$

Ans: $y = \frac{5x}{27} + 10$

Calculate the derivative of each given function, ie find y'

9- y=$9x^3 - 3x^2 + 2$

Ans: $27x^2 - 6x$

10- y=$4x^3 + 4x^2 + 2x$

Ans: $12x^2 + 8x + 2$.

11- y= $2x^{5/3} - 8x^{-2} - 12$

Ans: $\frac{10}{3}x^{\frac{2}{3}} + 16x^{-3}$

12- y= $7y^4 + 4y^{4/5}$

Ans: $28y^3 + \frac{16}{5}y^{-\frac{1}{5}}$

13-y=$3e^4 + 2x^7$

Ans: $14x^6$

14- y=$6e^x + 4x^3$

Ans: $6e^x + 12x^2$

3.3- Product and Quotient Rules

1-Use the Product Rule to calculate the derivative of $f(x) = x^3(3x^2 + 7)$

2-Use the Product Rule to calculate $h'(9)$ when. $h(s)=\left(s^{-1/2}+2s\right)\left(7-s^{-1}\right)$

3- Find y'(3) when. $y=\dfrac{1}{x+2}$

Use the Quotient or Product Rule to calculate the derivative for each function

4- $f(x)=\dfrac{3x}{4x-2}$

5- $f(x)=\dfrac{x+11}{x^2+4x+1}$

6- $f(x)=\dfrac{10}{1+e^x}$

7-$f(x) = (7x + 1)(x^2 - 2)$.

8-$f(x) = x^8(5 + x^{-1})$.

9- $f(x)=\dfrac{x^2-7}{x-4}$

10- $f(x)=\dfrac{x^3+2x^2+4x^{-1}}{x}$

11- $f(x)=(\sqrt{x}+5)(\sqrt{x}-2)$

12- $f(x)=\dfrac{x^6+e^x}{x+1}$

13- $f(x)=\dfrac{\cos(t)}{t}$

14-$f(x) = -\csc(x) - \sin(x)$

16-$f(x) = 8e^x \cos x$

17- $f(x)=\dfrac{x^2+7x+3}{\sqrt{x}}$

18- Find the second derivative of the function. $f(x) = x\cos(x)$, ie., find the derivative of f' , after you find f' from f.

19- The velocity of an object in meters per second is $v(t) = 42 - t^2$, $0 \le t \le 6$. Find the velocity $v(5)$ and acceleration $a(5)$ of the object when $t = 5$. What can be said about the speed of the object when the velocity and acceleration have opposite signs?

20-The curve $y = \dfrac{1}{x^2+1}$ is called the *witch of Agnesi* after the Italian mathematician Maria Agnesi (1718−1799) who wrote one of the first books on calculus. This strange name is the result of a mistranslation of the Italian word la versiera, meaning "that which turns." Find equations of the tangent lines to the curve at $x = \pm 3$.

3.3- Product and Quotient Rules solutions

1-Use the Product Rule to calculate the derivative of $f(x) = x^3(3x^2 + 7)$

Ans: $f'(x) = x^3(6x) + (3x^2 + 7)3x^2 = 6x^4 + 9x^4 + 21x^2 = 15x^4 + 21x^2$

2-
Use the Product Rule to calculate $h'(9)$ when. $h(s) = \left(s^{-1/2} + 2s\right)\left(7 - s^{-1}\right)$

Ans: $h'(9) = 1124/81$

3- Find $y'(3)$ when. $y = \dfrac{1}{x+2}$

Ans: $y'(3) = \dfrac{1}{(x+2)^2}$ so $y'(3) = -1/25$

Use the Quotient or Product Rule to calculate the derivative for each function

4- $f(x) = \dfrac{3x}{4x-2}$

Ans: $-\dfrac{6}{(4x-2)^2}$

5- $f(x) = \dfrac{x+11}{x^2+4x+1}$

Ans: $\dfrac{-x^2 - 22x - 43}{(x^2+4x+1)^2}$

6- $f(x) = \dfrac{10}{1+e^x}$

Ans: $\dfrac{-10e^x}{(e^x+1)^2}$

7-$f(x) = (7x + 1)(x^2 - 2)$.

Ans:
$f'(x) = 21x^2 + 2x - 14$

8-$f(x) = x^8(5 + x^{-1})$.

Ans:

$f'(x) = 40x^7 + 7x^6$

9- $f(x) = \dfrac{x^2 - 7}{x - 4}$

Ans: $\dfrac{x^2 - 8x + 7}{(x-4)^2}$

10- $f(x) = \dfrac{x^3 + 2x^2 + 4x^{-1}}{x}$

Ans: $2x + 2 - \dfrac{8}{x^3}$

11- $f(x) = (\sqrt{x} + 5)(\sqrt{x} - 2)$

Ans: $1 + \dfrac{3}{2\sqrt{x}}$

12- $f(x) = \dfrac{x^6 + e^x}{x + 1}$

Ans: $\dfrac{(e^x + 6x^5)(x+1) - e^x - x^6}{(x+1)^2}$

13- $f(x) = \dfrac{\cos(t)}{t}$

Ans: $-\dfrac{\sin(t) \cdot t + \cos(t)}{t^2}$

14-f$(x) = -\csc(x) - \sin(x)$

Ans: $\csc(x) \cdot \cot(x) - \cos(x)$

16-f$(x) = 8e^x \cos x$

Ans: $8 \cdot e^x(-\sin(x) + \cos(x))$

17- $f(x) = \dfrac{x^2 + 7x + 3}{\sqrt{x}}$

Ans : $\dfrac{3x^2 + 7x - 3}{2x^{\frac{3}{2}}}$

18- Find the second derivative of the function. $f(x) = x\cos(x)$

Ans: $-2 \cdot \sin(x) - x \cdot \cos(x)$

19- The velocity of an object in meters per second is $v(t) = 42 - t^2$, $0 \le t \le 6$. Find the velocity $v(5)$ and acceleration $a(5)$ of the object when $t = 5$. What can be said about the speed of the object when the velocity and acceleration have opposite signs?

Ans:

$$\begin{aligned} v(t) &= 42 - t^2,\ 0 \le t \le 6 \\ a(t) &= v'(t) = -2t \\ v(5) &= 17 \text{ m/sec} \\ a(5) &= -10 \text{ m/sec}^2 \end{aligned}$$

The speed of the object is decreasing.

20-The curve $y = \dfrac{1}{x^2+1}$ is called the *witch of Agnesi* after the Italian mathematician Maria Agnesi (1718−1799) who wrote one of the first books on calculus. This strange name is the result of a mistranslation of the Italian word la versiera, meaning "that which turns." Find equations of the tangent lines to the curve at $x = \pm 3$.

Ans:

For $x = -3$ and $x = 3$, we have respectively the equations

$$\frac{3}{50}(x+3) + \frac{1}{10}$$

$$-\frac{3}{50}(x-3) + \frac{1}{10}$$

3.4 The chain rule

Find the derivative of each function.

1-$y = 4(7 - x^2)^3$

2-$f(t) = (9t + 18)^{2/3}$

3-$\sqrt{2-7t}$

4-$y = \dfrac{6x}{\sqrt{x^2+4}}$

5-$g(\theta) = (\sin(2\theta))^8$

6-$y = \cos^4(\theta + 6)$

7-$y = \cos(2\theta + 41)$

8-$y = (2\cos(\theta + 34) + 5\sin(\theta + 34))^9$

9- $y = \sqrt{9 + 5x + sin(5x)}$

10-$y = e^{12x - 12}$

11-$f(x) = (x^5 - x^3 - 1)^{2/9}$

12-$y = \tan(6\theta^2 - 28\theta)$

13-$y = e^{2x^8}$

14-$y = e^{(4x^2+3x+9)^2}$

15-$y = e^{2x^8}$

Given the functions $f(x)$ and $g(x)$, find $f(g(x))$, $f'(x)$, $f'(g(x))$, $g'(x)$, and $(f \circ g)'$

16- $f(x) = x^3$ and $g(x) = 2x + 5$

17- $f(x) = x^5 + x$ and $g(x) = \cos(6x)$.

3.4 The chain rule solutions

Find the derivative of each function.

1-$y = 4(7 - x^2)^3$

Ans:

$y' = 4(3)(7 - x^2)^2(-2x) = -24x(7 - x^2)^2$

2-$f(t) = (9t + 18)^{2/3}$

Ans: $\dfrac{6}{(9 \cdot t + 18)^{\frac{1}{3}}}$

3-$\sqrt{2 - 7t}$

Ans: $-\dfrac{7}{2\sqrt{2 - 7t}}$

4-$y = \dfrac{6x}{\sqrt{x^2 + 4}}$

Ans: $\dfrac{24}{\sqrt{(x^2 + 4)^3}}$

5-$g(\theta) = (\sin(2\theta))^8$

Ans: $16\left(\sin(2\theta)\right)^7 \cos(2\theta)$

6-$y = \cos^4(\theta + 6)$

Ans: $-4\cos^3(\theta + 6)\sin(\theta + 6)$

7-$y = \cos(2\theta + 41)$

Ans: $-2\sin(2\theta + 41)$

8-$y = (2\cos(\theta + 34) + 5\sin(\theta + 34))^9$

Ans: $9\left(2\cos(\theta + 34) + 5\sin(\theta + 34)\right)^8 \left(5\cos(\theta + 34) - 2\sin(\theta + 34)\right)$

9- $y = \sqrt{9 + 5x + \sin(5x)}$

Ans: $\dfrac{5 + 5\cos(5x)}{2\sqrt{9 + 5x + \sin(5x)}}$

10-$y = e^{12x - 12}$

Ans: $12e^{12x-12}$

11-$f(x) = (x^5 - x^3 - 1)^{2/9}$

Ans: $\frac{2}{9}\left(x^5 - x^3 - 1\right)^{-\frac{7}{9}}\left(5x^4 - 3x^2\right)$

12- $y = \tan(6\theta^2 - 28\theta)$

Ans: $(\sec^2(6\theta^2 - 28\theta))(12\theta - 28) = (12\theta - 28)\sec^2(6\theta^2 - 28\theta)$.

13- $y = e^{2x^8}$

Ans: $y' = e^{2x^8}(16x^7) = 16x^7 e^{2x^8}$.

14- $y = e^{(4x^2+3x+9)^2}$

Ans: $\frac{dy}{dx} = e^{(4x^2+3x+9)^2} \cdot 2(4x^2+3x+9)(8x+3) = 2(8x+3)(4x^2+3x+9)e^{(4x^2+3x+9)^2}$.

15- $y = e^{2x^8}$

Ans: $y' = e^{2x^8}(16x^7) = 16x^7 e^{2x^8}$.

Given the functions $f(x)$ and $g(x)$, find $f(g(x))$, $f'(x)$, $f'(g(x))$, $g'(x)$, $(f \circ g)'$

16- $f(x) = x^3$ and $g(x) = 2x + 5$

Ans:

$f(g(x)) = (2x + 5)^3$

$f'(x) = 3x^2$

$f'(g(x)) = 3(2x + 5)^2$

$g'(x) = 2$

$(f \circ g)' = 6(2x + 5)^2$

17- $f(x) = x^5 + x$ and $g(x) = \cos(6x)$.

Ans:

$f(g(x)) = \cos^5(6x) + \cos(6x)$

$f'(x) = 5x^4 + 1$

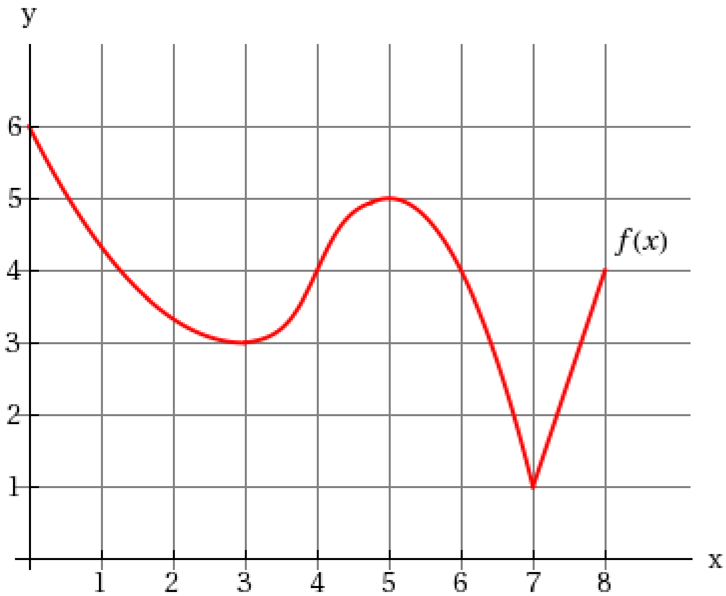
$f'(g(x)) = 5\cos^4(6x) + 1$

$g'(x) = -6\sin(6x)$

$(f \circ g)'(x) = -6\sin(6x) - 30\cos^4(6x)\sin(6x)$

4.3-4.4 First and Second Derivative test Problems with solutions (at the end)

1-Consider the function below.



(a)How many critical points does $f(x)$ have on [4, 8]?

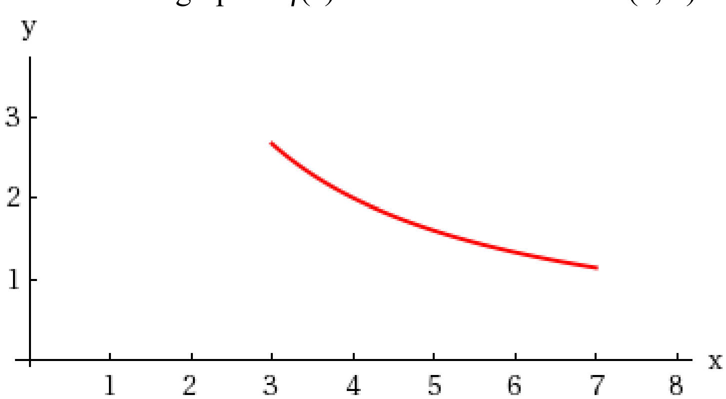
(b) What is the maximum value of $f(x)$ on [0, 6]?

(c) What is the local minimum value of $f(x)$ on [2, 5]?

(d) Find a closed interval on which both the minimum and maximum values of $f(x)$ occur at critical points.

(e) Find an interval on which the minimum value occurs at an endpoint

2-
Consider the graph of $f(x) = 8x^{-1}$ for this interval (3, 7).



Does $f(x) = 8x^{-1}$ have a local minimum, a local maximum or neither?

3-Find all critical values of the function. $f(x) = 8x^2 + 5x + 6$

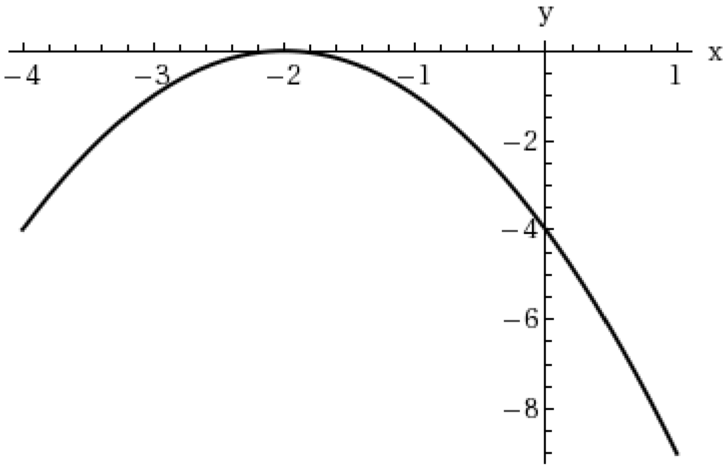
4-Find all critical values of the function. $f(x) = 9x - 2$

5-Find all critical values of the function. $f(x) = x^3 - 6x^2 - 36x + 21$

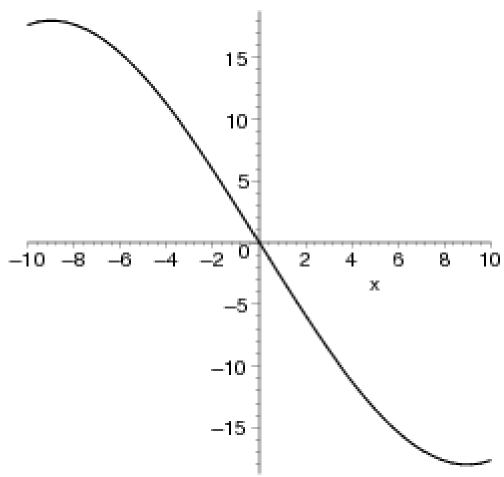
6-Find all critical values of the function. $f(x) = x \ln(5x)$.

7-Find all critical and the extreme values of the function $f(x) = 2x^3 - 9x^2 + 12x$ on the interval $[0, 3]$

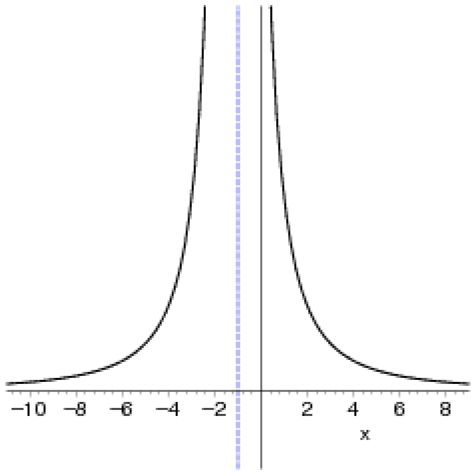
8- Use the graph to estimate the open intervals on which the function is increasing or decreasing. Then find these open intervals analytically. $y = -(x + 2)^2$



9-Use the graph to estimate the open intervals on which the function is increasing or decreasing. Then find the open intervals analytically. $y = \dfrac{x^3}{81} - 3x$



10-Use the graph to estimate the open intervals on which the function is increasing or decreasing. Then find the open intervals analytically. $f(x) = \dfrac{1}{(x+1)^2}$



11-Identify the open intervals on which the function is increasing or decreasing.
$g(x) = x^2 - 6x - 40$

12-Identify the open intervals on which the function is increasing or decreasing.
$f(x) = \sin(x) - 9, \quad 0 < x < 2\pi$

13-Identify the open intervals on which the function is increasing or decreasing.
$f(x) = x + 2\sin(x), \quad 0 < x < 2\pi$

14-Consider the following function. $f(x) = x^4 - 32x + 9$

(a) Find the critical numbers of f.
(b) Find the open intervals on which the function is increasing or decreasing.
(c) Apply the First Derivative Test to identify the relative extremum

15-Consider the following function. $f(x) = x^{1/3} + 9$

(a)Find the critical numbers of f.
(b) Find the open intervals on which the function is increasing or decreasing.
(c)Apply the First Derivative Test to identify the relative extremum

16-Consider the following function. $f(x) = 1 - |x - 9|$

(a)Find the critical numbers of f.
(b) Find the open intervals on which the function is increasing or decreasing.
(c)Apply the First Derivative Test to identify the relative extremum

17-Determine the open intervals on which the graph is concave upward or concave downward.
$y = 4x^2 - x - 2$

18-Determine the open intervals on which the graph is concave upward or concave downward.
$y = -x^3 - 6x^2 - 2$

19-Determine the open intervals on which the graph is concave upward or concave downward.
$g(x) = -6x^2 - x^3$

20-Determine the open intervals on which the graph is concave upward or concave downward.

$$f(x) = \frac{x^2 + 6}{x^2 - 1}$$

21- Determine the open intervals on which the graph is concave upward or concave downward.

$$y = 4x + \frac{2}{\sin x}, (-\pi, \pi)$$

22-Find the point of inflection of the graph of the function. Describe the concavity

$$f(x) = x^3 - 3x^2 + 17x$$

23-Find the points of inflection of the graph of the function. Describe the concavity.

$$f(x) = x\sqrt{3 - x}$$

Key:

1-(a) $f(x)$ has 2 critical points on the interval $[4, 8]$: at $x = 5$, $x = 7$.
(b) The maximum value of $f(x)$ on the interval $[0, 6]$ is 6; the function takes this value at $x = 0$.
(c) The local minimum value of $f(x)$ on the interval $[2, 5]$ is 3; the function takes this value at $x = 3$.
(d) On the interval $[2,6]$ both the minimum and maximum values of occur at critical points.
(e)For instance, on the interval $[0,2]$ the minimum value occurs at an endpoint.

2-$f(x)$ has no local minima or maxima. Hence, $f(x)$ only takes minimum and maximum values on an interval if it takes them at the endpoints. $f(x)$ takes no minimum or maximum value on this interval, since the interval does not contain its endpoints.

3-Let $f(x) = 8x^2 + 5x + 6$.Then $f'(x) = 16x + 5 = 0$ implies that $x = -5/16$ is the lone critical point of f.

4-Let $f(x) = 9x - 2$. Then $f'(x) = 9$, which is never zero, so $f(x)$ has **no** critical points.

5-Let $f(x) = x^3 - 6x^2 - 36x + 21$. Then $f'(x) = 3x^2 - 12x - 36 = 3(x + 2)(x - 6) = 0$ implies that $x = -2$ and $x = 6$ are the critical points of f.

6-Since $f(x)$ is differentiable in its domain, the only critical points of $f(x)$ are the solutions to $f'(x) = 0$.The derivative is $f'(x) = \ln(5x) + 1$.Setting this equal to zero, we find a solution by raising e to both sides of the equation, yielding $5x = e^{-1}$ or $x = e^{-1}/5$

7- Let $f(x) = 2x^3 - 9x^2 + 12x$. Then $f'(x) = 6x^2 - 18x + 12 = 0$ which yields $x = 1$ or $x = 2$.
Next, we compare:

x -value	**value of $f(x)$**
1 (critical point)	5
2 (critical point)	4
0 (endpoint)	0
3 (endpoint)	9

Thus, the minimum is 0, and the maximum is 9.

8- $y = -(x + 2)^2$
From the graph, f is increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$.
Analytically, $y' = -2(x + 2)$. Therefore, the critical number is $x = -2$. No discontinuity.

Test Intervals:	$-\infty < x < -2$	$-2 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$
Conclusion:	Increasing	Decreasing

9- $y=\frac{x^3}{81}-3x$ From the graph, y is increasing on $(-\infty, -9)$ and $(9, \infty)$, and decreasing on $(-9, 9)$.

Analytically, $y'=\frac{3x^2}{81}-3=\frac{3(x^2-81)}{81}=\frac{3}{81}(x^2-81)$

Critical numbers: $x = \pm 9$

Test Intervals:	$-\infty < x < -9$	$-9 < x < 9$	$9 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Increasing

10-From the graph, f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$.

Analytically, $f'(x)=\frac{-2}{(x+1)^3}$

No critical numbers. Discontinuity: $x=-1$

Test Intervals:	$-\infty < x < -1$	$-1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

11-

$g(x) = x^2 - 6x - 40$

$g'(x) = 2x - 6$

Critical number: $x = 3$

Test Intervals:	$-\infty < x < 3$	$3 < x < \infty$
Sign of $g'(x)$:	$g' < 0$	$g' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(3, \infty)$
Decreasing on: $(-\infty, 3)$

12-

$f(x) = \sin x + 9, \quad 0 < x < 2\pi$ and $f'(x) = \cos x$

Critical numbers: $x=\frac{\pi}{2},\frac{3\pi}{2}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

13-

$y = x + 2\sin(x), \quad 0 < x < 2\pi$

$y' = 1 + 2\cos(x)$

$y' = 0$ implies $\cos(x) = -1/2$

Critical numbers: $x =$ $x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

Test intervals:	$0 < x < \dfrac{2\pi}{3}$	$\dfrac{2\pi}{3} < x < \dfrac{4\pi}{3}$	$\dfrac{4\pi}{3} < x < 2\pi$
Sign of y':	$y' > 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Increasing

14-

(a) $f(x) = x^4 - 32x + 9$

$f'(x) = 4x^3 - 32 = 4(x^3 - 8)$

Critical number: $x = 2$

(b)

Test intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(2, \infty)$

Decreasing on: $(-\infty, 2)$

(c) Relative minimum: (2, -39)

15-

$f(x) = x^{1/3} + 9, \quad 0 < x < 2\pi$

$f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

(a) Critical number: $x = 0$ (since $f'(x)$ is undefined)

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' > 0$
Conclusion:	Increasing	Increasing

Increasing on: $(-\infty, \infty)$

Decreasing on: nowhere

(c) No relative extremum.

16-

(a) $f(x) = 1 - |x - 9|$, so $f'(x) = -\dfrac{x-9}{|x-9|} = \begin{cases} 1 & x < 9 \\ -1 & x > 9 \end{cases}$

Critical number: $x = 9$

(b)

Test intervals:	$-\infty < x < 9$	$9 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 9)$
Decreasing on: $(9, \infty)$

(c) Relative maximum: $(9, 1)$

17-

$y = 4x^2 - x - 2$
$y' = 8x - 1$
$y'' = 8$
Concave upward: $(-\infty, \infty)$

18-

$y = -x^3 - 6x^2 - 2$
$y' = -3x^2 - 12x$
$y'' = -6x - 12$
Concave upward: $(-\infty, -2)$
Concave downward: $(-2, \infty)$

19-

$g(x) = -6x^2 - x^3$
$g'(x) = -12x - 3x^2$
$g''(x) = -12 - 6x$
Concave upward: $(-\infty, -2)$
Concave downward: $(-2, \infty)$

20-

$$f(x) = \frac{x^2+6}{x^2-1} \Rightarrow f'(x) = \frac{-14x}{(x^2-1)^2} \Rightarrow f''(x) = \frac{14\left(3x^2+1\right)}{(x^2-1)^3}$$

Concave upward: $(-\infty, -1)$, $(1, \infty)$
Concave downward: $(-1, 1)$

21-

$y = 4x + 2\csc(x), \quad (-\pi, \pi)$

$y' = 4 - 2\csc(x)\cot(x)$

$y'' = -2\csc(x)(-(\csc(x))^2) - 2\cot(x)(-\csc(x)\cot(x))$

$= 2((\csc(x))^3 + \csc(x)(\cot(x))^2)$

Concave upward: $(0, \pi)$

Concave downward: $(-\pi, 0)$

22-

$f(x) = x^3 - 3x^2 + 17x$

$f'(x) = 3x^2 - 6x + 17$

$f''(x) = 6(x - 1) = 0$ when $x = 1$.

Concave upward: $(1, \infty)$

Concave downward: $(-\infty, 1)$

Point of inflection: $(1, 15)$

23-

$f(x) = x\sqrt{3-x} \qquad Domain: x \le 3$

$f'(x) = \dfrac{3(2-x)}{3\sqrt{3-x}} \Rightarrow f''(x) = \dfrac{3(x-4)}{4(3-x)^{3/2}}$

Concave downward: $(-\infty, 3)$

No point of inflection

4.5 Limits at Infinity (asymptotic behavior)

1-Consider the following limit. $f(x)=\dfrac{10x+1}{5x-7}$

a-Complete the table. (Round your answers to five decimal places.)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

b-Use the table to determine $\lim\limits_{x\to\infty} f(x)$

2-Consider the following limit. $f(x)=\dfrac{9x^2}{x+8}$

a-Complete the table. (Round your answers to one decimal place.)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

b-Use the table to determine $\lim\limits_{x\to\infty} f(x)$

3-Consider the following limit. $f(x)=9+\dfrac{1}{x^2+1}$

a-Complete the table. (Round your answers to one decimal place.)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

b-Use the table to determine $\lim\limits_{x\to\infty} f(x)$

4-Find the given limits, if possible. $f(x) = 9x^2 + 2x + 1$ find:

(a) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x}\right)$

(b) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x^2}\right)$

(c) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x^3}\right)$ or use l'Hopital rule

(d) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x^n}\right), n\geq 3$

Find each limit (use the l'Hopital rule when possible) in the next few questions (5-21)- **Note that if** $\lim\limits_{x\to\pm\infty} f(x) = c$ **then** y **= c is called a horizontal asymptote.**

5-

(a) $\lim\limits_{x\to\infty}\left(\dfrac{x^9+7}{x^{10}-9}\right)$

(b) $\lim\limits_{x\to\infty}\left(\dfrac{x^9+7}{x^9-9}\right)$

(c) $\lim\limits_{x\to\infty}\left(\dfrac{x^9+7}{x^8-9}\right)$

6-

(a) $\lim\limits_{x\to\infty}\left(\dfrac{5-8x}{5x^3-3}\right)$

(b) $\lim\limits_{x\to\infty}\left(\dfrac{5-8x}{5x-3}\right)$

(c) $\lim\limits_{x\to\infty}\left(\dfrac{5-8x^2}{5x-3}\right)$

7-

(a) $\lim\limits_{x\to\infty}\left(\dfrac{9x^{3/2}}{2x^3+1}\right)$

(b) $\lim\limits_{x\to\infty}\left(\dfrac{9x^{3/2}}{2x^{3/2}+1}\right)$

(c) $\lim\limits_{x\to\infty}\left(\dfrac{9x^{3/2}}{2\sqrt{x}+1}\right)$

8- $\lim\limits_{x\to\infty}\left(9+\dfrac{2}{x}\right)$

9- $\lim\limits_{x\to\infty}\left(\dfrac{9}{x}-\dfrac{x}{2}\right)$

10- $\lim\limits_{x\to\infty}\left(\dfrac{x}{\sqrt{x^2-x}}\right)$

11- $\lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{9x-1}\right)$

12- $\lim_{x\to-\infty}\left(\frac{\sqrt{x^{10}-1}}{x^9-1}\right)$

13- $\lim_{x\to\infty}\left(\frac{2}{9x+\sin x}\right)$

14- $\lim_{x\to\infty} 9\cos(\frac{9}{x})$

15- $\lim_{x\to\infty}\frac{\sin(9x)}{x}$

16- $\lim_{x\to\infty}\frac{3x-\cos(x)}{x}$

17- $\lim_{x\to\infty}\log_6(1+6^{-x})$

18- $\lim_{x\to\infty}\left[\frac{5}{2}+\ln\left(\frac{x^2+6}{x^2}\right)\right]$

19- $\lim_{x\to\infty} x\sin\frac{3}{x}$ (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$, or use l'Hopital rule)

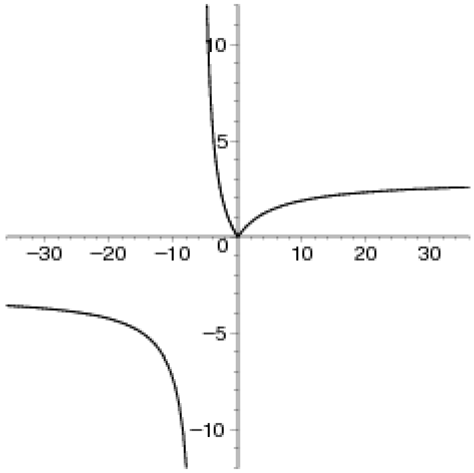
20- $\lim_{x\to\infty} 2x\tan\left(\frac{5}{x}\right)$ (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$, or use l'Hopital rule)

21- $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2}\right)$ (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.)

21- $\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)$ (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.)

22-Consider the graph of the function

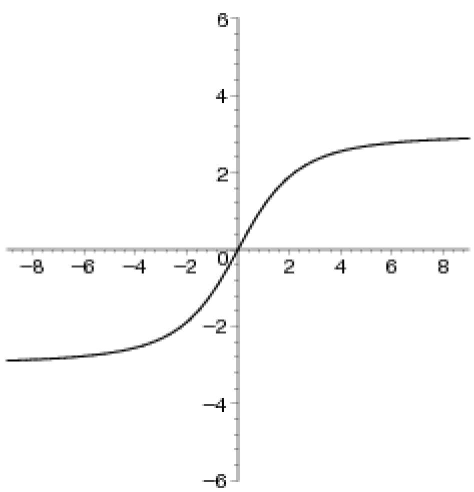
$f(x) = \dfrac{3|x|}{x+6}$



Find the equations of any horizontal asymptotes. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

23-Consider the graph of the function

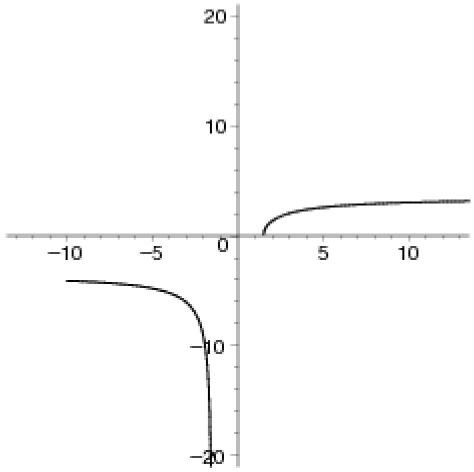
$f(x) = \dfrac{3x}{\sqrt{x^2+6}}$



Find the equations of any horizontal asymptotes. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

23- Consider the graph of the function

$f(x) = \dfrac{\sqrt{49x^2-110}}{2x+3}$



Find the equations of any horizontal asymptotes. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

- **You will not be asked to sketch graphs, for this course even though it is good for practice!**

24-Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes.
$x^2y = 2$

25-Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes.

$$y = \frac{3x}{x-2}$$

26- (a) Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes.

$$y = \frac{1}{x^2 - 4x - 32}$$

(b) Find any extrema that exist.

(c) Find any vertical asymptote(s)

(d) Find any horizontal asymptote(s)

4.5 Limits at Infinity (asymptotic behavior) Solutions

1-Consider the following limit. $f(x) = \dfrac{10x+1}{5x-7}$

a-Complete the table. (Round your answers to five decimal places.)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	-5.50000	2.34884	2.03043	2.00300	2.00030	2.00003	2.00000

b-Use the table to determine $\lim\limits_{x\to\infty} f(x)$

Ans: 2

2-Consider the following limit. $f(x) = \dfrac{9x^2}{x+8}$

a-Complete the table. (Round your answers to one decimal place.)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	1.0	50.0	833.3	8,928.6	89,928.1	899,928.0	8,999,928.0

b-Use the table to determine $\lim\limits_{x\to\infty} f(x)$

Ans: ∞

3-Consider the following limit. $f(x) = 9 + \dfrac{1}{x^2+1}$

a-Complete the table. (Round your answers to one decimal place.)

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	9.50000	9.00990	9.00010	9.00000	9.00000	9.00000	9.00000

b-Use the table to determine $\lim\limits_{x\to\infty} f(x)$

Ans: 9

4-Find the given limits, if possible. $f(x) = 9x^2 + 2x + 1$ find:

(a) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x}\right)$

Ans: ∞

(b) $\lim\limits_{x\to\infty}\left(\dfrac{f(x)}{x^2}\right)$

Ans: 9

(c) $\lim_{x\to\infty}\left(\frac{f(x)}{x^3}\right)$

Ans: 0

(d) $\lim_{x\to\infty}\left(\frac{f(x)}{x^n}\right), n \geq 3$

Ans: 0 -- *f(x)* and x^n are said to be **asymptotically equivalent**, for all $n > 2$

Find each limit (use l'Hopital rule when possible) in the next few questions (5-21)- **Note that if** $\lim_{x\to\pm\infty} f(x) = c$ **then *y* = c is called a horizontal asymptote.**

5-

(a) $\lim_{x\to\infty}\left(\frac{x^9+7}{x^{10}-9}\right)$

Ans: 0

(b) $\lim_{x\to\infty}\left(\frac{x^9+7}{x^9-9}\right)$

Ans: 1

(c) $\lim_{x\to\infty}\left(\frac{x^9+7}{x^8-9}\right)$

Ans: ∞

6-

(a) $\lim_{x\to\infty}\left(\frac{5-8x}{5x^3-3}\right)$

Ans: 0

(b) $\lim_{x\to\infty}\left(\frac{5-8x}{5x-3}\right)$

Ans: -8/5

(c) $\lim_{x\to\infty}\left(\frac{5-8x^2}{5x-3}\right)$

Ans: -∞

7-

(a) $\lim_{x\to\infty}\left(\frac{9x^{3/2}}{2x^3+1}\right)$

Ans: 0

(b) $\lim_{x\to\infty}\left(\frac{9x^{3/2}}{2x^{3/2}+1}\right)$

Ans: 9/2

(c) $\lim_{x\to\infty}\left(\frac{9x^{3/2}}{2\sqrt{x}+1}\right)$

Ans: ∞

8- $\lim_{x\to\infty}\left(9+\frac{2}{x}\right)$

Ans: 9

9- $\lim_{x\to\infty}\left(\frac{9}{x}-\frac{x}{2}\right)$

Ans: ∞

10- $\lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2-x}}\right)$

Ans:-1

11- $\lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{9x-1}\right)$

Ans:1/9

12- $\lim_{x\to-\infty}\left(\frac{\sqrt{x^{10}-1}}{x^9-1}\right)$

Ans: 0

13- $\lim_{x\to\infty}\left(\frac{2}{9x+\sin x}\right)$

Ans: 0

14- $\lim_{x\to\infty}9\cos(\frac{9}{x})$

Ans: 9

15- $\lim_{x\to\infty}\frac{\sin(9x)}{x}$

Ans:0

16- $\lim_{x\to\infty}\frac{3x-\cos(x)}{x}$
Ans:3

17- $\lim_{x\to\infty}\log_6(1+6^{-x})$

Ans: 0

18- $\lim_{x\to\infty}\left[\frac{5}{2}+\ln\left(\frac{x^2+6}{x^2}\right)\right]$

Ans: 5/2

19- $\lim_{x\to\infty}x\sin\frac{3}{x}$ (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$, or use l'Hopital rule)

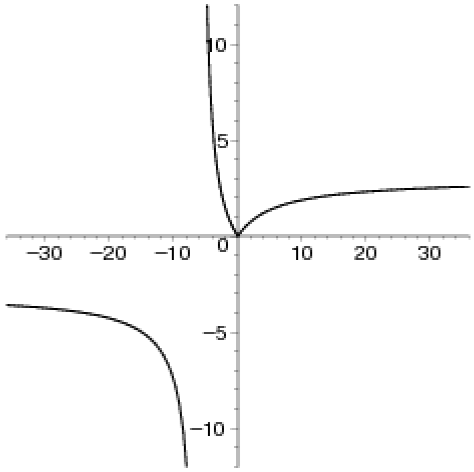
Ans: 3

20- $\lim_{x\to\infty}2x\tan\left(\frac{5}{x}\right)$ (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$, or use l'Hopital rule)
Ans: 10

21- $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2}\right)$ (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.)
Ans: 0

21- $\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)$ (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.)
Ans: 1/6

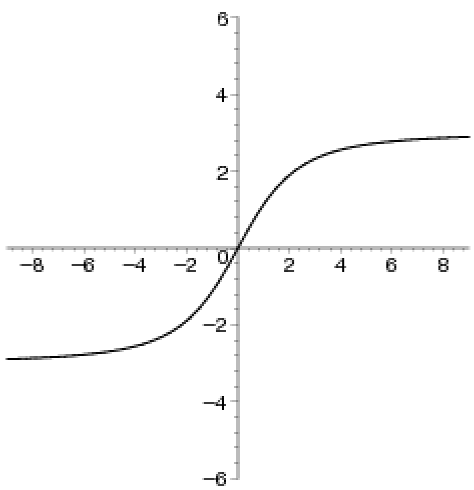
22-Consider the graph of the function
$f(x)=\frac{3|x|}{x+6}$



Find the equations of any horizontal asymptotes. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)
Ans: $y = -3, y = 3$

23-Consider the graph of the function

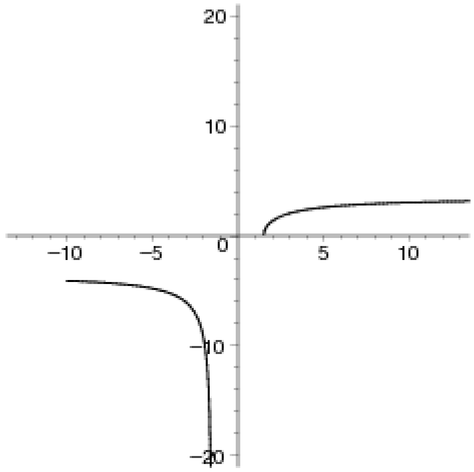
$f(x) = \dfrac{3x}{\sqrt{x^2 + 6}}$



Find the equations of any horizontal asymptotes. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)
Ans: $y = 3, y = -3$

23- Consider the graph of the function

$f(x) = \dfrac{\sqrt{49x^2 - 110}}{2x + 3}$

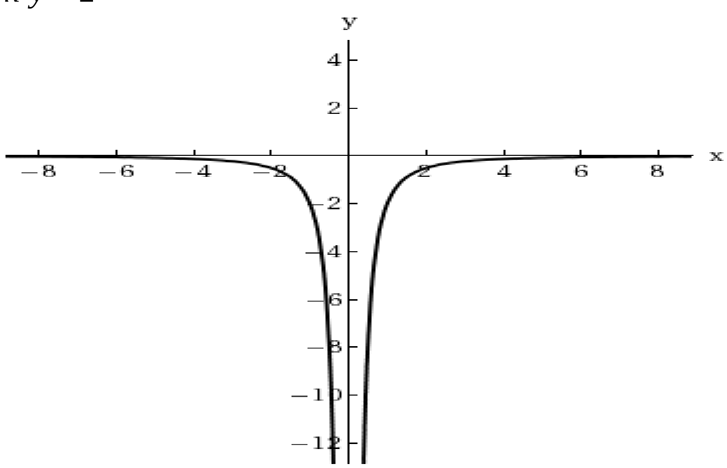


Find the equations of any horizontal asymptotes. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

Ans: $y = \frac{7}{2}, y = -\frac{7}{2}$

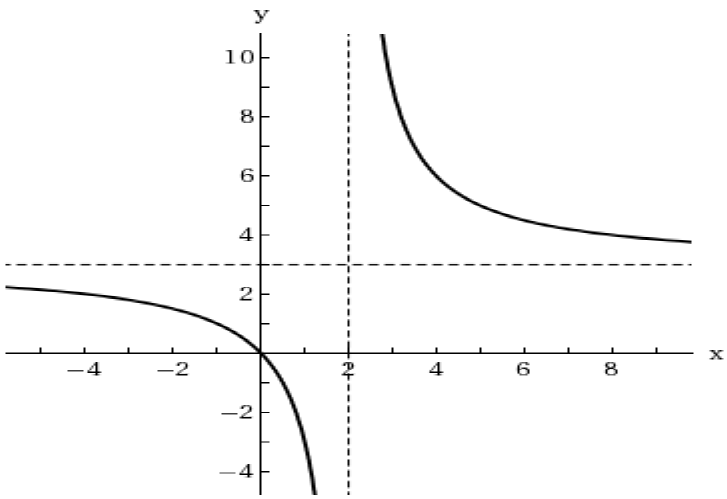
- **You will not be asked to sketch graphs, for this course even though it is good for practice!**

24-Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes.
$x^2y = 2$



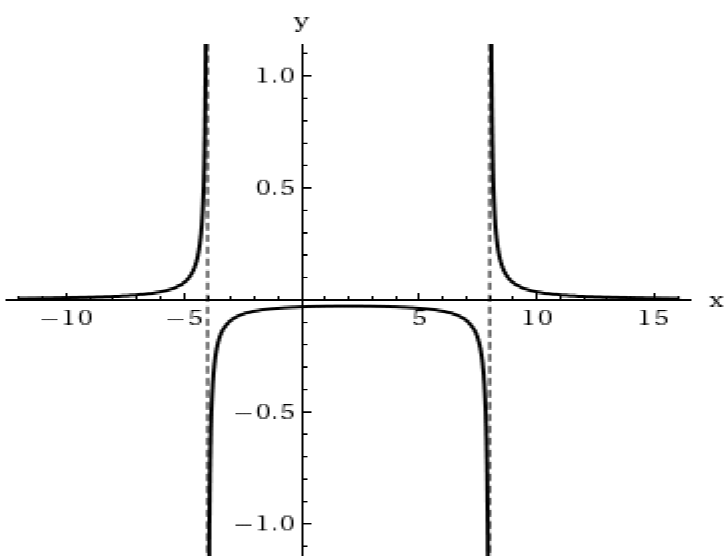
25-Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes.

$$y = \frac{3x}{x-2}$$



26-(a) Sketch the graph of the equation using extrema, intercepts, symmetry, and asymptotes.

$$y = \frac{1}{x^2 - 4x - 32}$$



(b) Find any extrema that exist.
Ans: relative maximum (2,-1/36); No relative minimum

(c) Find any vertical asymptote(s)
Ans: x=-4, 8

(d) Find any horizontal asymptote(s)
Ans: y=0

Linear Approximation 4.8 and Newton's Method 3.8

1- Find the equation of the tangent line *T* to the graph of *f* at the given point. Use this linear approximation to complete the table. (Round your answers to four decimal places.)
$f(x) = x^2$, (7, 49)

x	6.9	6.99	7	7.01	7.1
$f(x)$					
$T(x)$					

2- Find the equation of the tangent line *T* to the graph of *f* at the given point. Use this linear approximation to complete the table. (Round your answers to four decimal places.)
$f(x) = \frac{4}{x^2}, (4, \frac{1}{4})$

x	3.9	3.99	4	4.01	4.1
$f(x)$					
$T(x)$					

3-Find the equation of the tangent line *T* to the graph of *f* at the given point. Use this linear approximation to complete the table. (Round your answer to four decimal places.)
$f(x) = \sqrt{x}, (3, \sqrt{3})$

x	2.9	2.99	3	3.01	3.1
$f(x)$					
$T(x)$					

4-Find the equation of the tangent line *T* to the graph of *f* at the given point. Use this linear approximation to complete the table. (Round your answers to four decimal places.)
$f(x) = \sin x$, (2, sin 2)

x	1.9	1.99	2	2.01	2.1
$f(x)$					
$T(x)$					

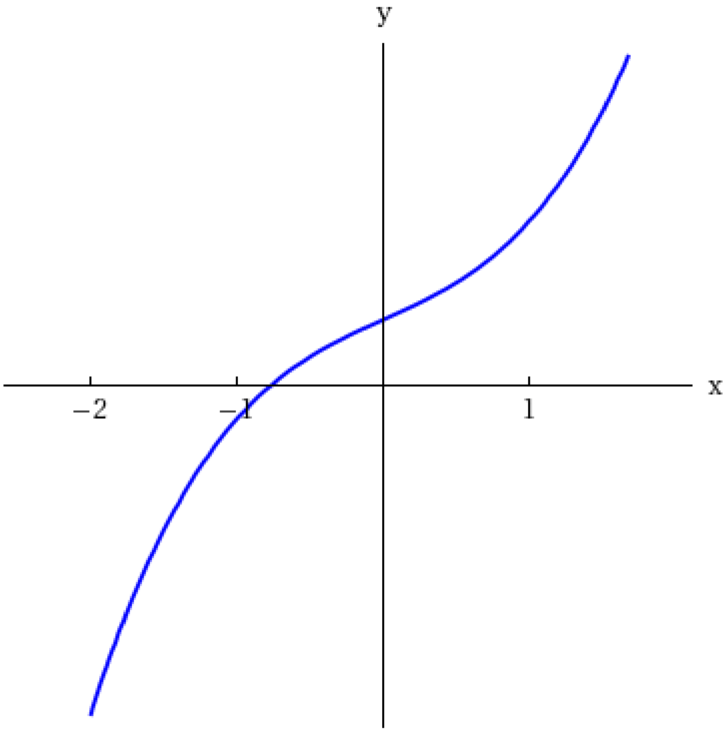
5-Apply Newton's Method to $f(x)$ and initial guess
x_0 to calculate x_1, x_2, and x_3.(Round your answers to seven decimal places.)
$f(x) = x^2 - 12$, $x_0 = 3$

6-Apply Newton's Method to $f(x)$ and initial guess
x_0 to calculate x_1, x_2, and x_3.(Round your answers to seven decimal places.)
$f(x) = x^2 - 7x + 2$, $x_0 = 7$

7-Apply Newton's Method to $f(x)$ and initial guess
x_0 to calculate x_1, x_2, and x_3.(Round your answers to seven decimal places.)
$f(x) = x^3 + 17x + 1$, $x_0 = -1$

8-Apply Newton's Method to $f(x)$ and initial guess
x_0 to calculate x_1, x_2, and x_3.(Round your answers to seven decimal places.)
$f(x) = 1 - 9x \sin x$, $x_0 = 7$

9-The figure below is the graph of $f(x) = x^3 + 2x + 2$
Use Newton's method to approximate the unique real root of
$f(x) = 0$.(Round your answer to five decimal places.)



10-The ninth positive solution of $\sin x = 0$ is $x = 9\pi$.
Use Newton's Method to calculate the first three approximations of 9π.
(Let $x_0 = 28$. Round your answers to five decimal places.)

11-Use Newton's Method to calculate the first three approximations to the root
$7^{1/3}$. Let $x_0 = 2$. (Round your answers to five decimal places.)

Linear Approximation 4.8 and Newton's Method 3.8 Solutions

1-

$f(x) = x^2$

$f'(x) = 2x$

Tangent line at (7, 49): $y - f(7) = f'(7)(x - 7)$

$y - 49 = 14(x - 7)$

$y = 14x - 49$

x	6.9	6.99	7	7.01	7.1
$f(x) = x^2$	47.6100	48.8601	49.0000	49.1401	50.4100
$T(x) = 14x - 49$	47.6000	48.8600	49.0000	49.1400	50.4000

2-

$$f(x) = \frac{4}{x^2}$$

$$= 4x^{-2}$$

$f'(x) = -8x^{-3} = \dfrac{-8}{x^3}$. Now, tangent line at $(4, \frac{1}{4})$: $T(x): y = \dfrac{-1}{8}x + \dfrac{3}{4}$

x	3.9	3.99	4	4.01	4.1
$f(x)$	0.2630	0.2513	0.2500	0.2488	0.2380
$T(x)$	0.2625	0.2513	0.2500	0.2488	0.2375

3- $f(x) = \sqrt{x} \Rightarrow f'(x) = \dfrac{1}{2\sqrt{x}}$

Now, tangent line at $(3, \sqrt{3})$

$$y = f(3) = f'(3)(x-3)$$

$$y - \sqrt{3} = \frac{1}{2\sqrt{3}}(x-3)$$

$$T(x): y = \frac{x}{2\sqrt{3}} + \frac{\sqrt{3}}{2}$$

x	2.9	2.99	3	3.01	3.1
$f(x)$	1.7029	1.7292	1.7321	1.7349	1.7607
$T(x)$	1.7032	1.7292	1.7321	1.7349	1.7609

4-

$f(x) = \sin x$

$f'(x) = \cos x$

Tangent line at (2, sin 2):

$y - f(2) = f'(2)(x - 2)$

$y - \sin 2 = (\cos 2)(x - 2)$

$y = (\cos 2)(x - 2) + \sin 2$

x	1.9	1.99	2	2.01	2.1
$f(x) = \sin(x)$	0.9463	0.9134	0.9093	0.9051	0.8632
$T(x) = (\cos 2)(x - 2) + \sin 2$	0.9509	0.9135	0.9093	0.9051	0.8677

5-

Let $f(x) = x^2 - 12$ and define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^{\ 2} - 12}{2x_n}$

With $x_0 = 3$,we compute

$x_1 \approx 3.5$

$x_2 \approx 3.4642857$

$x_3 \approx 3.4641016$.

6-

Let $f(x) = x^2 - 7x + 2$ and define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^{\ 2} - 7x_n + 2}{2x_n - 7}$

With $x_0 = 7$,we compute

$x_1 \approx 6.7142857$

$x_2 \approx 6.7015873$

$x_3 \approx 6.7015621$.

7-

Let $f(x) = x^3 + 17x + 1$and define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^{\ 3} + 17x_n + 1}{3x_n^{\ 2} + 17}$

With $x_0 = -1$,we compute

$x_1 \approx -0.15$

$x_2 \approx -0.0589864$

$x_3 \approx -0.0588116$.

8-

Let $f(x) = x^3 + 17x + 1$and define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{1 - 9x_n \sin(x_n)}{-9x_n \cos(x_n) - 9\sin(x_n)}$

With $x_0 = 7$,we compute

$x_1 \approx 6.2437536$

$x_2 \approx 6.3013793$

$x_3 \approx 6.3008207$.

9-

Let $f(x) = x^3 + 2x + 2$ and define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^{\ 3} + 2x_n + 2}{3x_n^{\ 2} + 2}$

We take x_0= -0.8, based on the figure, and then calculate

$x_1 \approx -0.77143$

$x_2 \approx -0.77092$

$x_3 \approx -0.77092$

The root, to five decimal places, is -0.77092.

10-

Let $f(x) = \sin x$ and define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{\sin(x_n)}{\cos(x_n)}$

Taking $x_0 = 28$, we have

$x_1 \approx 28.28143$

$x_2 \approx 28.27433$

$x_3 \approx 28.27433$.

11-

Let $f(x) = x^3 - 7$, since its root is $7^{1/3.}$

Now, define $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^{\ 3} - 7}{3x_n^{\ 2}}$

Taking $x_0 = 2$, we have

$x_1 \approx 1.91667$

$x_2 \approx 1.91294$

$x_3 \approx 1.91293$

5.1 Antideratives and Indefinite Integration

1-Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.)

$$\frac{dy}{dt} = 45t^4$$

2-Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.)

$$\frac{dy}{dt} = 4x^{4/9}$$

3-Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.)

$$\frac{dy}{dt} = 4x^{-5}$$

For the next questions (4-17), find the indefinite integral and check the result by differentiation. (Use C for the constant of integration.)

4- $\int (x^{3/2} + 6x + 9)dx$

5- $\int (\sqrt{x} + \frac{1}{5\sqrt{x}})dx$

6- $\int (\sqrt[4]{x^3})dx$

7- $\int (\frac{1}{x^5})dx$

8- $\int (\frac{x+4}{\sqrt{x}})dx$

9- $\int \left(4\ cosx + \ 9\ sinx\right)dx$

10- $\int (\theta^4 + \ sec^2\theta)d\theta$

11- $\int \left(4sinx - 9e^x\right)dx$

12- $\int \left(sec(y)tan(y) \ - \ (csc(y))^2 \right) dy$

13- $\int \left(6x - 9^x\right) dx$

14- $\int \left(6x - 9^x\right) dx$

15- $\int \left(\cos x - 4^x\right) dx$

16- $\int \left(5x - \frac{9}{x}\right) dx$

17- $\int \left(\frac{4}{x} + \sec^2 x\right) dx$

18- Find the particular solution that satisfies the differential equation and the initial condition.
$f'(x) = 8x, \quad f(0) = 5$

19- Find the particular solution that satisfies the differential equation and the initial condition.
$f'(s) = 14s - 4s^3, \quad f(3) = 1$

20- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = 4, \quad f'(2) = 10, \quad f(2) = 16$

21- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = x^2, \quad f'(0) = 8, \quad f(0) = 4$

22- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = x^{-3/2}, \quad f'(4) = 7, \quad f(0) = 0$

23-Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = \sin(x), \quad f'(0) = 8, \quad f(0) = 9$

24- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = e^x, \quad f'(0) = 5, \quad f(0) = 7$

25- An evergreen nursery usually sells a certain shrub after 8 years of growth and shaping. The growth rate during those 8 years is approximated by $dh/dt = 1.4t + 2$, where t is the time in years and h is the height in centimeters. The seedlings are 19 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.

(b) How tall are the shrubs when they are sold?

26- The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days ($0 \leq t \leq 10$) as shown below.

$$\frac{dP}{dt} = k\sqrt{t}$$

The initial size of the population is 700. After 1 day the population has grown to 800. Estimate the population after 6 days. (Round your answer to the nearest whole number.)

27- Use $a(t) = -32$ ft/sec^2 as the acceleration due to gravity. (Neglect air resistance.)

A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 62 feet per second. How high will the ball go? (Round your answer to two decimal places.)

28- Consider a particle moving along the x-axis where $x(t)$ is the position of the particle at time t, $x'(t)$ is its velocity, and $x''(t)$ is its acceleration.

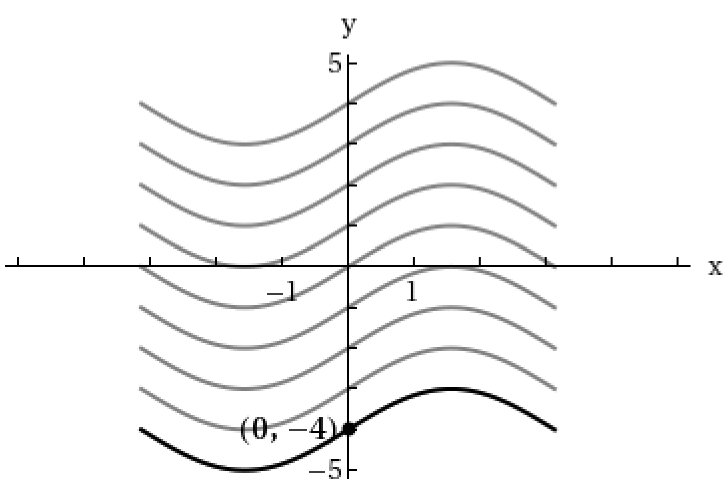
$x(t) = t^3 - 12t^2 + 21t - 5, \quad 0 \le t \le 10$

(a) Find the velocity and acceleration of the particle.

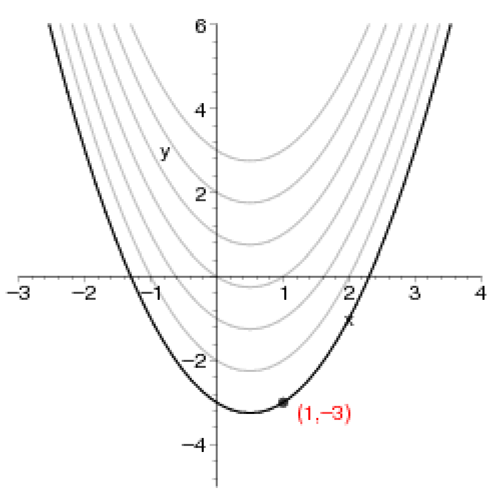
(b) Find the open t-intervals on which the particle is moving to the right. (Enter your answer using interval notation.)

(c) Find the velocity of the particle when the acceleration is 0.

29-Find the equation of y, given the derivative and the given point on the curve. $\dfrac{dy}{dx} = \cos x$.



30-Find the equation of y, given the derivative and the indicated point on the curve. $\dfrac{dy}{dx} = 2x - 1$



5.1 Antideratives and Indefinite Integration solutions

1-Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.)

$$\frac{dy}{dt} = 45t^4$$

$y = C + 9t^5$

2-Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.)

$$\frac{dy}{dt} = 4x^{4/9}$$

$y = \frac{36}{13} \cdot x^{\frac{13}{9}} + C$

3-Find the general solution of the differential equation and check the result by differentiation. (Use C for the constant of integration.)

$$\frac{dy}{dt} = 4x^{-5}$$

$y = C - \frac{1}{x^4}$

For the next questions (4-17), find the indefinite integral and check the result by differentiation. (Use C for the constant of integration.)

4-

$$\int (x^{3/2} + 6x + 9)dx$$

$y = \frac{2}{5} \cdot x^{\frac{5}{2}} + 3 \cdot x^2 + 9 \cdot x + C$

5-

$$\int (\sqrt{x} + \frac{1}{5\sqrt{x}})dx$$

$y = \frac{2}{15} \cdot x^{\frac{1}{2}} \cdot (5 \cdot x + 3) + C$

6-

$$\int (\sqrt[4]{x^3})dx$$

$y= C + \frac{4x^{7/4}}{7}$

7-

$\int (\frac{1}{x^5})dx$

$y= \frac{-1}{4x^4} + C$

8-

$\int (\frac{x+4}{\sqrt{x}})dx$

Ans: $y= \frac{2}{3} \cdot x^{\frac{1}{2}} \cdot (12+x) + C$

9- $\int (4\ cosx + \ 9\ sinx)\, dx$

Ans: $C + 4\sin(x) - 9\cos(x)$

10- $\int (\theta^4 + \ sec^2\theta)d\theta$

Ans: $C + \frac{\theta^5}{5} + \tan(\theta)$

11- $\int (4sinx - 9e^x)\, dx$

Ans: $C - 9e^x - 4\cos(x)$

12- $\int \left(sec(y)tan(y) \ - \ (csc(y))^2 \right) dy$

Ans: $\sec(y) + \cot(y) + C$

13- $\int (6x - 9^x)\, dx$

Ans: $C + 3x^2 - \frac{9^x}{\ln(9)}$

14- $\int (6x - 9^x)\, dx$

Ans: $C + 3x^2 - \frac{9^x}{\ln(9)}$

15- $\int \left(\cos x - 4^x \right) dx$

Ans: $C + \frac{4^x}{\ln(4)} + \sin(x)$

16- $\int \left(5x - \frac{9}{x} \right) dx$

Ans: $-9\ln(|x|) + C + \frac{5x^2}{2}$

17- $\int \left(\frac{4}{x} + \sec^2 x \right) dx$

Ans: $4\ln(|x|) + C + \tan(x)$

18- Find the particular solution that satisfies the differential equation and the initial condition.
$f'(x) = 8x, \quad f(0) = 5$
Ans: $f(x) = 4x^2 + 5$

19- Find the particular solution that satisfies the differential equation and the initial condition.
$f'(s) = 14s - 4s^3, \quad f(3) = 1$
Ans: $f(s) = 7s^2 - 1s^4 + 19$

20- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = 4, \quad f'(2) = 10, \quad f(2) = 16$
Ans: $f(x) = 2 \cdot x^2 + 2 \cdot x + 4$

21- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = x^2, \quad f'(0) = 8, \quad f(0) = 4$
Ans: $f(x) = \frac{1}{12}x^4 + 8x + 4$

22- Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = x^{-3/2}, \quad f'(4) = 7, \quad f(0) = 0$
Ans: $f(x) = -4\sqrt{x} + 8x$
23-Find the particular solution that satisfies the differential equation and the initial condition.
$f''(x) = \sin(x), \quad f'(0) = 8, \quad f(0) = 9$
$f(x) = -\sin(x) + 9 \cdot x + 9$

24- Find the particular solution that satisfies the differential equation and the initial condition.

$f''(x) = e^x$, $f'(0) = 5$, $f(0) = 7$
Ans: $f(x) = 4x + e^x + 6$

25- An evergreen nursery usually sells a certain shrub after 8 years of growth and shaping. The growth rate during those 8 years is approximated by $dh/dt = 1.4t + 2$, where t is the time in years and h is the height in centimeters. The seedlings are 19 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.
Ans: $h(t) = 0.7t^2 + 2t + 19$

(b) How tall are the shrubs when they are sold?
Ans:79.8 cm

26- The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days ($0 \le t \le 10$) as shown below.

$$\frac{dP}{dt} = k\sqrt{t}$$

The initial size of the population is 700. After 1 day the population has grown to 800. Estimate the population after 6 days. (Round your answer to the nearest whole number.)
Ans: 2170 bacteria

27- Use $a(t) = -32$ ft/sec^2 as the acceleration due to gravity. (Neglect air resistance.)

A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 62 feet per second. How high will the ball go? (Round your answer to two decimal places.)

Ans: 66.06 ft

28- Consider a particle moving along the x-axis where $x(t)$ is the position of the particle at time t, $x'(t)$ is its velocity, and $x''(t)$ is its acceleration.
$x(t) = t^3 - 12t^2 + 21t - 5, \quad 0 \le t \le 10$

(a) Find the velocity and acceleration of the particle.
Ans: respectively,

$3t^2 - 24t + 21$

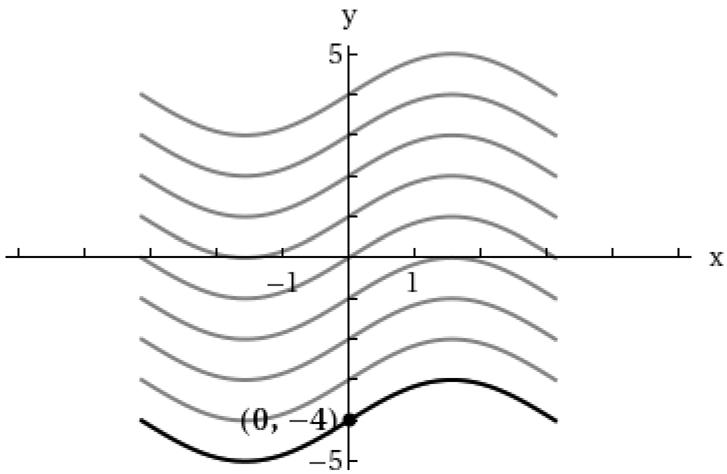
$6(t - 4)$

(b) Find the open t-intervals on which the particle is moving to the right. (Enter your answer using interval notation.)
$(0, 1) \cup (7, 10)$

(c) Find the velocity of the particle when the acceleration is 0.

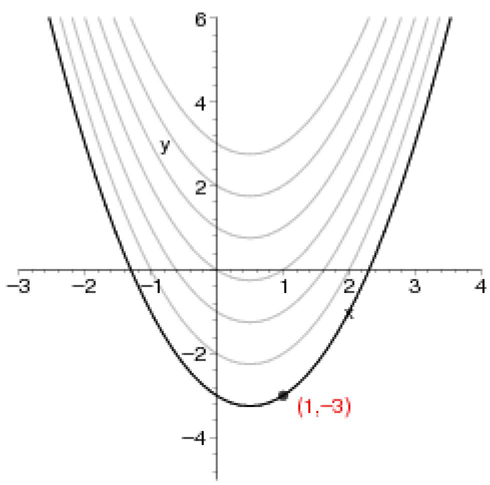
Ans: -27

29-Find the equation of y, given the derivative and the given point on the curve. $\dfrac{dy}{dx} = \cos x$.



Ans:y= $\sin(x) - 4$

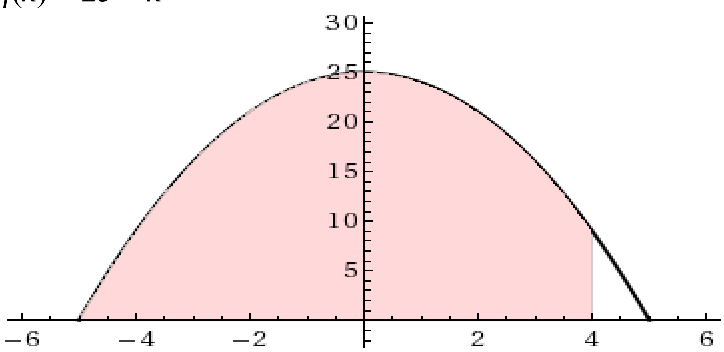
30-Find the equation of *y*, given the derivative and the indicated point on the curve. $\dfrac{dy}{dx} = 2x - 1$



Ans: y=$x^2 - x - 3$

5.3-5.4 Definite Integrals and Fundamental Theorem of Calculus

1-Set up a definite integral that yields the area of the region. (Do not evaluate the integral.)
$f(x) = 25 - x^2$

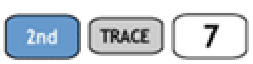


Ans: $\int_{-5}^{4} 25 - x^2 dx$

- Evaluate each definite integral (NO DECIMAL) and verify your answer by using a graphing utility with the steps given below (a-d).

For instance, for the definite integral $\int_2^9 \left(\frac{8}{x^2} - 1\right) dx$

a. enter the function $y = \frac{8}{x^2} - 1$

b. Press 

, to evaluate the integral.

c. Enter the lower bound as 2.

d. Enter the upper bound as 9.

The calculator displays the following screen.

$\int_2^9 f(x)\,dx = -3.8888889$ which actual value is -35/9

2- $\int_{-2}^{2}\left(t^2-4t\right)dt$

Ans: -32/3

3- $\int_{0}^{4}\left(4t-1\right)^2 dt$

Ans: 844/3

5- $\int_{5}^{8}\left(\frac{7}{x^2}-1\right)dx$

Ans: -99/40

6- $\int_{1}^{9}\left(\frac{u-7}{\sqrt{u}}\right)du$

Ans: -32/3

7- $\int_{1}^{36}\left(\sqrt{\frac{6}{x}}\right)dx$

Ans: $10\sqrt{6}$

8- $\int_{0}^{1}\left(\frac{x-\sqrt{x}}{6}\right)dx$

Ans: -1/36

9- $\int_{0}^{3}\left((3-t)\sqrt{t}\right)dt$

Ans: $\frac{12\sqrt{3}}{5}$

10- $\int_{-1}^{0}\left(t^{1/3}-t^{2/3}\right)dt$

Ans: -27/20

11- $\int_{0}^{7}\left|2x-7\right|dx$

Ans: 49/2

12- $\int_{0}^{10} \left| x^2 - 8x + 7 \right| dx$

Ans: 226/3

13- $\int_{2}^{8} \left(6 - \left| x - 6 \right|\right) dx$

Ans: 26

14- $\int_{0}^{\pi} \left(6 + \sin x\right) dx$

Ans: 2+6π

15- $\int_{-\pi/6}^{\pi/6} \left(5\sec^2 x\right) dx$

Ans: $\dfrac{10\sqrt{3}}{3}$

16- $\int_{\pi/4}^{\pi/2} \left(2 - cs\,\mathrm{c}^2\, x\right) dx$

Ans: $\dfrac{-2+\pi}{2}$

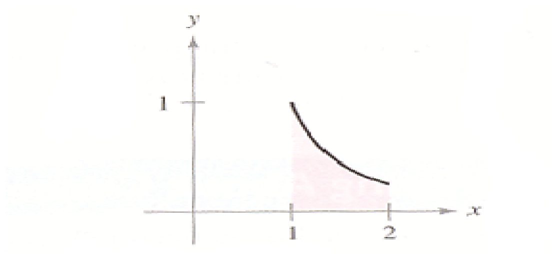
17- $\int_{0}^{4} \left(2^x + 5\right) dx$

Ans: $20 + \dfrac{15}{\ln(2)}$

18- $\int_{-6}^{6} \left(e^{\theta} + \sin\theta\right) d\theta$

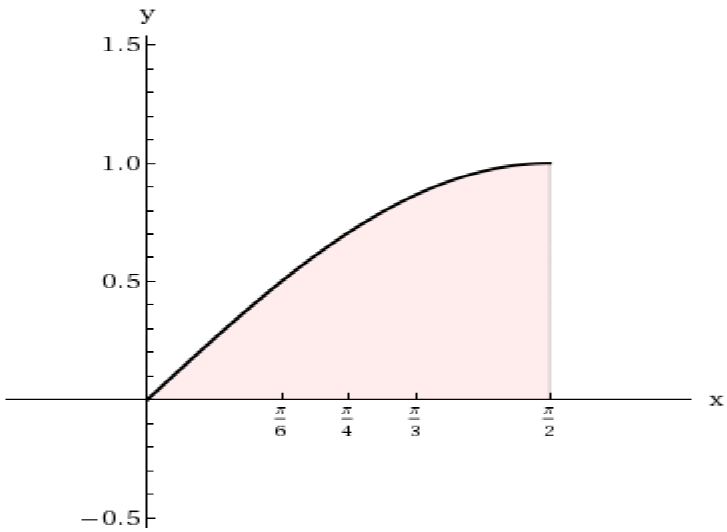
Ans: $e^6 - \dfrac{1}{e^6}$

20-Determine the area of the given region under the curve. $y = \dfrac{1}{x^6}$



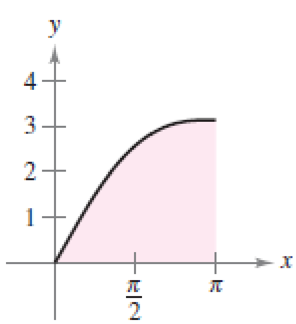
Ans: 31/160

21- Determine the area of the given region.
$y = \sin x$



Ans: 1

22-Determine the area of the given region.
$y = x + \sin x$



Ans: $\frac{\pi^2+4}{2}$

23- Find the area of the region bounded by the graphs of the equations.

$y = 5x^2 + 3,\ x = 0,\ x = 2,\ y = 0$

Ans: $\frac{58}{3}$

24-Find the area of the region bounded by the graphs of the equations. $y = 6 + \sqrt[3]{x}$, $x = 0$, $x = 8$, $y = 0$

Ans: 60

25- Find the area of the region bounded by the graphs of the equations. $y = \frac{6}{x}$, $x = 1$, $x = e$, $y = 0$

And: 6

26-Find the area of the region bounded by the graphs of the equations.
$y = e^x$, $x = 0$, $x = 6$, $y = 0$

Ans: $e^6 - 1$

27-Use the Second Fundamental Theorem of Calculus to find $F'(x)$. $F(x) = \int_{-3}^{x} \sqrt{t^4 + 7}\,dt$

Ans: $F'(x) = \sqrt{x^4 + 7}$

28-Use the Second Fundamental Theorem of Calculus to find $F'(x)$. $F(x) = \int_{0}^{\sin(x)} 7\sqrt{t}\,dt$

Ans: $F'(x) = 7\sqrt{\sin(x)}\cos(x)$

29-Use the Second Fundamental Theorem of Calculus to find $F'(x)$. $F(x) = \int_{0}^{x^5} \sin t^3\,dt$

Ans: $F'(x) = 5x^4 \sin(x^{15})$

30-Use the Second Fundamental Theorem of Calculus to find $F'(x)$. $F(x)=\int_{x}^{x^5}\frac{\sin t}{1-\ln t}dt, x>0$

Ans: $F'(x)=\frac{\sin x^5}{1-\ln x^5}5x^4-\frac{\sin x}{1-\ln x}$

31-Use the Second Fundamental Theorem of Calculus to find $F'(x)$. $F(x)=\int_{\sqrt{x}}^{1}\tan t^2 dt$

Ans: $F'(x)=-\frac{\tan x}{2\sqrt{x}}$

5.5 Substitution technique

1- Find the indefinite integral and check the result by differentiation. $\int (4+3x)^4 (3)\,dx$

2- Find the indefinite integral and check the result by differentiation. $\int \sqrt[3]{(4-5x^2)}(-10x)\,dx$

3-Find the indefinite integral and check the result by differentiation. $\int x^2 (x^3+2)^9\,dx$

4- Find the indefinite integral and check the result by differentiation. $\int x(6x^2+4)^3\,dx$

5-Find the indefinite integral and check the result by differentiation. $\int t^2\sqrt{t^3+4}\,dt$

6- Find the indefinite integral and check the result by differentiation. $\int \frac{x}{(8-2x^2)^3}\,dx$

7- Find the indefinite integral $\int \left(2x^2+\frac{1}{(4x^2)^2}\right)dx$

8- Find the indefinite integral $\int \frac{t-2t^4}{\sqrt{t}}\,dt$

9- Find the indefinite integral $\int 4\pi y(7+y^{5/2})\,dy$

10- Find the indefinite integral $\int \frac{1}{\theta^2}\cos\left(\frac{1}{\theta}\right)d\theta$

11-Solve the differential equation. $\frac{dy}{dx}=\frac{x-2}{\sqrt{x^2-4x+4}}$

12-Find the indefinite integral. $\int \sec(8-x)\tan(8-x)\,dx$

13-Find the indefinite integral. $\int \sqrt{\tan(6x)}\sec^2(6x)\,dx$

14-Find the indefinite integral. $\int \frac{-\cos x}{\sin^4 x}\,dx$

15-Find the indefinite integral. $\int \frac{e^{3x}+3e^x+2}{e^x}\,dx$

16- Find the indefinite integral. $\int \frac{1}{19-5x}\,dx$

17-Find the indefinite integral. $\int \frac{x^3 - 3x^2 + 9}{x - 3} dx$

18- Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \frac{x^3 - 2x^2 + 4x - 6}{x^2 + 3} dx$

19-Find the indefinite integral. $\int \frac{(\ln x)^2}{x} dx$

20-Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \frac{1}{x \ln x^2} dx$

21-Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \sec x dx$

22-Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \frac{\sec x \tan x}{\sec x - 8} dx$

5.5 Substitution technique solutions

1- Find the indefinite integral and check the result by differentiation. $\int (4+3x)^4 (3)\, dx$

Ans: $\frac{1}{5}\cdot(4+3\cdot x)^5 + C$

2- Find the indefinite integral and check the result by differentiation. $\int \sqrt[3]{(4-5x^2)}(-10x)\, dx$

Ans: $\frac{3}{4}(4-5x^2)^{4/3} + C$

3-Find the indefinite integral and check the result by differentiation. $\int x^2 \left(x^3+2\right)^9 dx$

Ans: $\frac{1}{30}\cdot\left(x^3+2\right)^{10} + C$

4- Find the indefinite integral and check the result by differentiation. $\int x\left(6x^2+4\right)^3 dx$

Ans: $\frac{1}{48}\cdot\left(6\cdot x^2+4\right)^4 + C$

5-Find the indefinite integral and check the result by differentiation. $\int t^2\sqrt{t^3+4}\,dt$

Ans: $\frac{2}{9}\left(t^3+4\right)^{3/2}$ + C

6- Find the indefinite integral and check the result by differentiation. $\int \frac{x}{(8-2x^2)^3}dx$

$\frac{1}{8\left(8-2x^2\right)^2} + C$

7- Find the indefinite integral $\int \left(2x^2 + \frac{1}{(4x^2)^2} \right) dx$

Ans: $\frac{2x^3}{3} - \frac{1}{16x} + C$

8- Find the indefinite integral $\int \frac{t - 2t^4}{\sqrt{t}} dt$

Ans: $\frac{2}{27} \cdot t^{\frac{3}{2}} \cdot (9 - 6 \cdot t^3) + C$

9- Find the indefinite integral $\int 4\pi y (7 + y^{5/2}) dy$

Ans: $\frac{8}{9} \pi y^{9/2} + 14\pi y^2 + C$

10- Find the indefinite integral $\int \frac{1}{\theta^2} \cos\left(\frac{1}{\theta} \right) d\theta$

Ans: $-\sin\left(\frac{1}{\theta} \right) + C$

11-Solve the differential equation. $\frac{dy}{dx} = \frac{x-2}{\sqrt{x^2 - 4x + 4}}$

Ans: $(x^2 - 4 \cdot x + 4)^{\frac{1}{2}} + C$

12-Find the indefinite integral.

$\int \sec(8 - x) \tan(8 - x) dx$

Ans: $-\sec(8 - x) + C$

13-Find the indefinite integral. $\int \sqrt{\tan(6x)} \sec^2(6x) dx$

Ans: $\frac{1}{9} \tan^{\frac{3}{2}}(6x) + C$

14-Find the indefinite integral. $\int \frac{-\cos x}{\sin^4 x} dx$

Ans: $\frac{\csc^3(x)}{3} + C$

15-Find the indefinite integral. $\int \frac{e^{3x} + 3e^x + 2}{e^x} dx$

Ans: $3x - 2e^{-x} + \frac{e^{2x}}{2} + C$

16- Find the indefinite integral. $\int \frac{1}{19 - 5x} dx$

Ans: $-\frac{1}{5} \cdot \ln(|-19 + 5 \cdot x|) + C$

17-Find the indefinite integral. $\int \frac{x^3 - 3x^2 + 9}{x - 3} dx$

Ans: $9\ln(|x - 3|) + \frac{x^3}{3}$ $+ C$

18- Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \frac{x^3 - 2x^2 + 4x - 6}{x^2 + 3} dx$

Ans: $-2 \cdot x + \frac{1}{2} \cdot x^2 + \frac{1}{2} \cdot \ln(x^2 + 3)$
$+ C$

19-Find the indefinite integral. $\int \frac{(\ln x)^2}{x} dx$

Ans: $\frac{\ln^3(x)}{3} + C$

20-Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \frac{1}{x \ln x^2} dx$

Ans: $\frac{1}{2}\ln(\ln(|x|)) + C$

21-Find the indefinite integral. (Remember to use ln(|*u*|) where appropriate.)

$\int \sec x dx$

Ans: ln|sec$x$+tan$x$|+C

22-Find the indefinite integral. (Remember to use ln(|u|) where appropriate.)

$$\int \frac{\sec x \tan x}{\sec x - 8} dx$$

Ans: $\ln(|\sec(x) - 8|) + C$

Numerical Integrations (5.6) and Area Approximations (5.2)

1- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^2 x^2 dx, \qquad n = 4$

2- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_1^2 \left(\frac{x^2}{3} + 8\right) dx, \qquad n = 4$

3- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_2^3 \frac{9}{x^6} dx, \qquad n = 4$

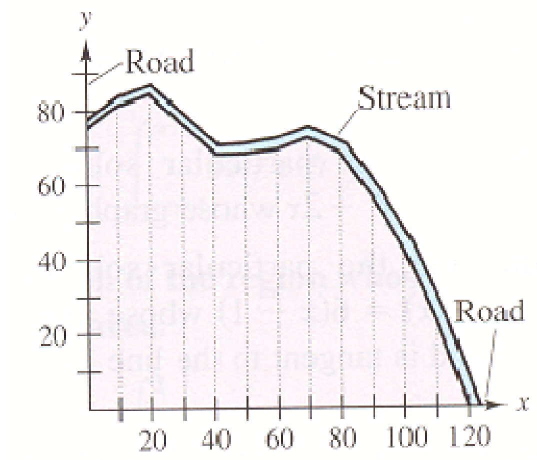
4- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^8 \sqrt[3]{x} dx, \qquad n = 8$

5- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^1 \frac{2}{(x+2)^2} dx, \qquad n = 4$

6- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^4 x\sqrt{x^2 + 4} dx, \qquad n = 4$

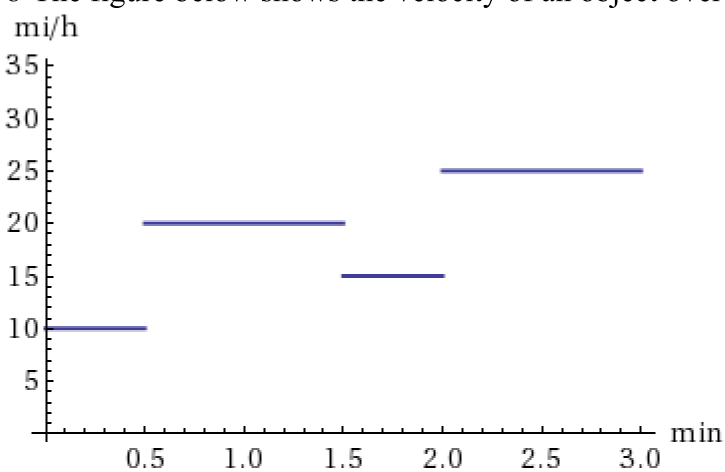
7-Use the Trapezoidal Rule to estimate the number of square meters of land in a lot where *x* and *y* are measured in meters, as shown in the figure. The land is bounded by a stream and two straight roads that meet at right angles. (Round your answer to the nearest whole number.)

x	0	10	20	30	40	50	60	70	80	90	100	110	120
y	75	80	85	76	66	68	70	71	67	57	43	23	0



Area Approximation and Riemann's Sum (5.2)

8-The figure below shows the velocity of an object over a 3-min interval.



a-Determine the distance traveled over the interval
[0, 3].
(Remember to convert from mi/hr to mi/min. Round your answer to two decimal places.)

b-Determine the distance traveled over the interval
[1, 2.5].(Remember to convert from mi/hr to mi/min. Round your answer to two decimal places.)

9-An animal runs with velocity 30 km/h for 4 minutes, 12 km/h for the next 2 minutes, and 25 km/h for another 5 minutes. Compute the total distance traveled. (Round your answer to two decimal places.)

10-A rainstorm hit a city resulting in record rainfall. The rainfall rate $R(t)$ is recorded, in centimeters per hour, in the following table, where t is the number of hours since midnight.

Compute the total rainfall during this 24-hour period.

t (h)	0−2	2−4	4−9	9−12	12−20	20−24
$R(t)$ (cm)	0.6	0.5	1.0	2.5	1.4	0.4

11-Compute R_5 and L_5 over [0, 1] using the following values.

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	50	47	46	45	44	41

12-Consider $f(x) = 5x + 2$ on [0, 3].
a-Compute L_6 and R_6 over [0, 3].

b-Use geometry to find the exact area A and compute the errors $L_6 - A$ and $R_6 - A$

13-Calculate R_8 for $f(x) = 6 - x$ over [3, 5].

14- Calculate the L_6 approximation for $f(x) = \sqrt{6(x+7)+2}$ on [1, 3].(Round your answer to six decimal places.)

15- Calculate the R_5 approximation for $f(x) = 11x^2 + 13x$ on [−1, 1].

16- Calculate the L_4 approximation for $f(x) = \cos^2 x$ on $\left[\frac{\pi}{8}, \frac{\pi}{2}\right]$ (Round your answer to four decimal places.)

Numerical Integrations (5.6) and Area Approximations (5.2) solutions

1- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^2 x^2 dx, \qquad n = 4$

Ans: Trapezoidal: 2.7500
Simpson's: 2.6667
Exact: 2.6667

2- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_1^2 \left(\frac{x^2}{3} + 8\right) dx, \qquad n = 4$

Ans: Trapezoidal: 8.7778

Simpson's: 8.7813

Exact: 8.7778

3- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_2^3 \frac{9}{x^6} dx, \qquad n = 4$

Ans: Trapezoidal: 0.0509
Simpson's: 0.0489
Exact: 211/4320

4- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of *n*. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^8 \sqrt[3]{x} dx, \qquad n = 8$

Ans: Trapezoidal: 11.7296
Simpson's: 11.8632
Exact: 12.0000

5- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of n. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^1 \frac{2}{(x+2)^2}dx, \qquad n=4$

Ans: Trapezoidal: 0.3352
Simpson's: 0.3334
Exact: 0.3333

6- Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for the given value of n. Round your answers to four decimal places and compare the results with the exact value of the definite integral. $\int_0^4 x\sqrt{x^2+4}dx, \qquad n=4$

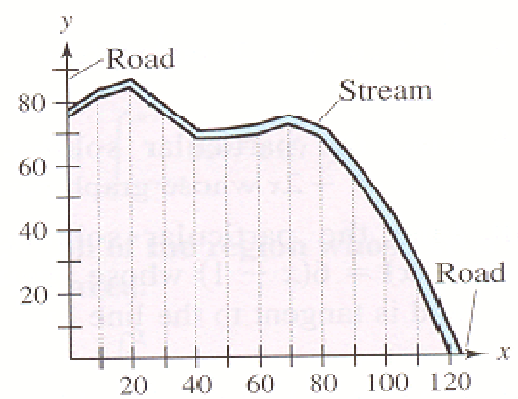
Ans: Trapezoidal: 27.6538
Simpson's: 27.1377
Exact: 27.1476

7-Use the Trapezoidal Rule to estimate the number of square meters of land in a lot where x and y are measured in meters, as shown in the figure. The land is bounded by a stream and two straight roads that meet at right angles. (Round your answer to the nearest whole number.)

Ans:

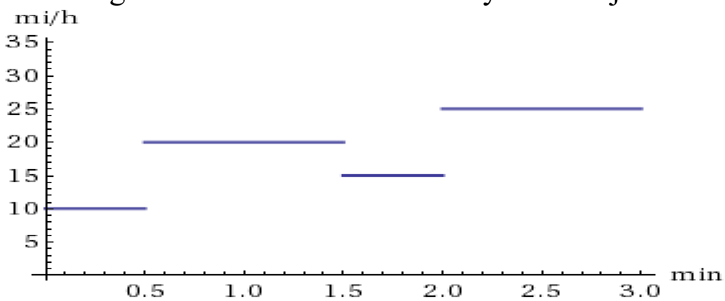
$$Area \approx \frac{120}{2(12)}\left[75+\ 2(80)\ +\ 2(85)\ +\ 2(76)\ +\ 2(66)\ +\ 2(68)\ +\ 2(70)\ +\ 2(71)\ +\ 2(67)\ +\ 2(57)\ +\ 2(43)\ +\ 2(23)\ +\ 0\right]$$
$$= 7435m^2$$

x	0	10	20	30	40	50	60	70	80	90	100	110	120
y	75	80	85	76	66	68	70	71	67	57	43	23	0



Area Approximation and Riemann's Sum (5.2)

8-The figure below shows the velocity of an object over a 3-min interval.



a-Determine the distance traveled over the interval
[0, 3].
(Remember to convert from mi/hr to mi/min. Round your answer to two decimal places.)
Ans: 0.96mi

b-Determine the distance traveled over the interval
[1, 2.5].(Remember to convert from mi/hr to mi/min. Round your answer to two decimal places.)
Ans: 0.50mi

9-An animal runs with velocity 30 km/h for 4 minutes, 12 km/h for the next 2 minutes, and 25 km/h for another 5 minutes. Compute the total distance traveled. (Round your answer to two decimal places.)
Ans:4.48 km

10-A rainstorm hit a city resulting in record rainfall. The rainfall rate $R(t)$ is recorded, in centimeters per hour, in the following table, where t is the number of hours since midnight. Compute the total rainfall during this 24-hour period.

t (h)	0−2	2−4	4−9	9−12	12−20	20−24
$R(t)$ (cm)	0.6	0.5	1.0	2.5	1.4	0.4

Ans: Over each interval, the total rainfall is the time interval in hours times the rainfall in centimeters per hour. Thus

$R = (0.6) \cdot 2 + (0.5) \cdot 2 + (1) \cdot 5 + (2.5) \cdot 3 + (1.4) \cdot 8 + (0.4) \cdot 4 = 27.5$centimeters.

11-Compute R_5 and L_5over [0, 1] using the following values.

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	50	47	46	45	44	41

Ans: $\Delta x = \frac{1-0}{5} = 0.2$ Thus,

$L_5 = 0.2 \cdot (50 + 47 + 46 + 45 + 44) = 46.4$ and

$R_5 = 0.2 \cdot (47 + 46 + 45 + 44 + 41) = 44.6$.

12-Consider $f(x) = 5x + 2$on [0, 3].

a-Compute L_6 and R_6 over [0, 3].

b-Use geometry to find the exact area A and compute the errors $L_6 - A$ and $R_6 - A$

Ans: We partition [0, 3] into 6 equally-spaced subintervals. The left endpoints of the subintervals are $\left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}\right\}$ whereas the right endpoints are $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right\}$

Let $a = 0$, $b = 3$, $n = 6$, $\Delta x = (b - a)/n = 0.5$, and $x_k = a + \Delta x$, $k = 0, 1, ..., 5$(left endpoints). Then

$L_6 = \sum_{k=0}^{5} f(x_k)\Delta x = \Delta x \sum_{k=0}^{5} f(x_k) = 0.5(2 + 4.5 + 7 + 9.5 + 12 + 14.5) = 24.75$

With $x_k = a + \Delta x$, $k = 1, 2, ..., 6$ (right endpoints), we have

$\sum_{k=0}^{5} f(x_k)\Delta x = \Delta x \sum_{k=0}^{5} f(x_k) =$ 0.5(4.5 + 7 + 9.5 + 12 + 14.5 + 17) = 32.25

Via geometry, the exact area is $A = 0.5(3)(15) + 2 \cdot 3 = 28.5$.

Thus, L_6 underestimates the true area ($L_6 - A = -3.75$), while R_6 overestimates the true area ($R_6 - A = 3.75$).

13-Calculate R_8 for $f(x) = 6 - x$ over [3, 5].

Ans: R_8=3.75

14- Calculate the L_6 approximation for $f(x) = \sqrt{6(x+7)+2}$ on [1, 3].

(Round your answer to six decimal places.)

Ans: $L_6 = 14.825231$

15- Calculate the R_5 approximation for $f(x) = 11x^2 + 13x$ on [−1, 1].

Ans: 328/25

16- Calculate the L_4 approximation for $f(x) = \cos^2 x$ on $\left[\frac{\pi}{8}, \frac{\pi}{2}\right]$ (Round your answer to four decimal places.)

Ans: $L_4 = 0.5431$